AN APPROACH TO THE FUZZY VARIABLE STRUCTURE CONTROL OF INDUCTION MOTORS

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ABSTRACT

This paper concerns fuzzy sliding mode control. A new approach, which was first introduced by Ben-Ghalia et al., is applied to the cascade sliding mode control of an induction motor fed by a PWM voltage source inverter, which operates in a fixed reference frame. For this purpose, a new decoupled and reduced model is first proposed. Then, a set of simple surfaces and associated control laws are synthesized. A piecewise smooth control function with a threshold is adopted. However, the magnitude of this function depends closely on the upper bound of uncertainties, which include parameter variations and external disturbances. This bound is difficult to obtain prior to motor operation. To solve this problem, a new fuzzy sliding mode control applied to an induction motor drive is presented. The fuzzy sliding mode controllers are designed in order to improve the control performances and to reduce the control energy and the chattering phenomenon. Simulation results reveal some very interesting features and show that the proposed fuzzy sliding mode controller could be considered as an alternative to the conventional sliding mode controllers of induction motors.

Key words: nonlinear feedback control, sliding mode control, Induction motor, cascade structure, fuzzy logic control, fuzzy sliding mode control
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1. INTRODUCTION

Induction motors are widely used in industry due to their reliability and relatively low cost compared with commonly used DC motors. However, induction motors are a theoretically challenging control problem since as a dynamical system they are nonlinear, the electric rotor variables are not measurable, and the model parameters are most often imprecisely known. The control of the induction motor has attracted much attention in the last two decades. One of the most significant developments in this area has been the feedback linearization control [1–3]. Partial feedback linearization together with a PI controller is used to regulate the motor state [2–4]. This technique is very useful except that it is very sensitive to parameter variation and unmodeled dynamics. To improve this type of control technique, full linearizing state feedback control based on differential geometric theory has been proposed for induction motor control. This method requires relatively complicated and nonlinear calculations in the control algorithm. When dealing with the control of uncertain nonlinear dynamical systems, conventional linear control design techniques linearize such nonlinear systems and assume a defined interval on the uncertainty in the system transfer function. The assumption of this interval does not, in general, reflect the actual uncertainty in the system parameters. In addition, the robustness of the controllers is valid only for operating conditions in the neighborhood of the operating point around which the system was linearized. To overcome some of these problems and to ensure the robustness of the feedback controller against parameter changes, unknown disturbances, initial conditions mismatch and load variation, different variable structure controls have been proposed.

The main objective of variable structure system (VSS) through sliding mode behavior is to force the system to reach a predetermined surface, known as the sliding manifold, and to constrain this trajectory to the sliding surface for all subsequent time via the use of appropriate switching logic. In general, the design of a sliding mode controller can be divided into two phases: the reaching phase and the sliding phase. By proper design of the sliding mode controller (SMC), the plant attains the conventional goals of control such as robustness, stabilization, and tracking [5, 6]. However, to guarantee the stability of the controlled systems, these control techniques require that the structure of uncertainty satisfy certain conditions. Such conditions are often referred to as matching conditions. Although matching conditions represent one way to handle uncertainty from a mathematical point of view, by providing tractable analytical solutions, it has been recognized that such conditions are very restrictive and are easily violated when practical applications are considered.

Recently, the application of fuzzy sets for the solution of control problems has been the focus of numerous studies [7–12]. The impetus behind this is often that the system knowledge and dynamic models are uncertain, and as an alternative to traditional modeling and control design, fuzzy set theory appears to provide a suitable representation of such knowledge and models. Traditionally, fuzzy controllers have been designed without an explicit model of the system. On the basis of this, the argument has been put forward that a nonlinear controller can be designed using linguistic qualitative knowledge rather than a mathematical model of the system. Fuzzy logic control (FLC) was first introduced and applied in the seventies [13, 14] in an attempt to design controllers for systems that are difficult to model. The methodology of the fuzzy sets appears very useful when the processes are too complex for analysis by conventional quantitative techniques, or when the available source of information is interpreted qualitatively, approximately, or uncertainly. Thus, fuzzy logic control may be viewed as a step toward a method for converting conventional precise mathematical control to human-like decision-making.

Since then, fuzzy logic control has become an active and fruitful research area with many theoretical works and industrial applications being reported [15, 18]. However, at the present time, there is no systematic procedure for the design of a fuzzy logic controller. The fundamentals of fuzzy logic control are still missing and the stability is not guaranteed. Applying notions of classical control theory to fuzzy logic control seems necessary.

It is well known that variable structure control can provide very robust performance [1, 5], however, due to insufficient mathematical tools, this comparison has only been qualitative. Furthermore, it is also well known that the performance of fuzzy logic control is not satisfactory on higher order systems. Many papers have dealt with learning and tuning of FLC [11, 12], however, a systematic method is still missing because the relationship between scaling gains and performance is not very clear.

Based on the above points, a fuzzy variable structure control approach, initially introduced by Ben-Ghali et al. [7–9], is applied for the design and tuning of fuzzy logic controllers with application to an induction motor. To shows the benefits of the proposed fuzzy sliding mode control algorithm, simulation results comparing the performance of the
proposed fuzzy model based control with that of conventional sliding mode control are presented. These controllers are evaluated by simulations for a variety of operating conditions of the drive system. The results obtained confirm that the proposed control structure improves the performance and the robustness of the drive system.

This paper is organized as follows.

The model of the motor based on feedback linearization is proposed in order to minimize the dependence of the system to parameter variations and external perturbations and is given in Section 2. Then, in order to minimize the dependence of the system to parameter variations and external perturbations, sliding mode controllers were investigated and their design is studied in Section 3, while the performance of such a control scheme is validated in Section 4. Furthermore, to overcome the problem of the chattering phenomenon, fuzzy logic controllers, one for each sliding mode controller, were implemented by using the approach of fuzzy sliding mode control proposed by Ben-Ghallia et al. [8, 9], as explained in Section 5. Finally, the validation of this fuzzy sliding control scheme is done in Section 6.

2. NONLINEAR FEEDBACK CONTROL

The state equations of the voltage PWM source inverter fed induction motor with current control, in a stator reference frame (\( \alpha-\beta \)), with \((i_{\alpha s}, i_{\beta s})\) as command variables and \((\phi_{\alpha r}, \phi_{\beta r}, \Omega)\) as state variables are given [19–22]:

\[
\dot{x} = f(x) + g(x) \cdot u
\]

where:

\[
x = (x_1, x_2, x_3)^T = (\Phi_{\alpha r}, \Phi_{\beta r}, \Omega)^T;
\]

\[
u = (u_1, u_2)^T = (i_{\alpha s}, i_{\beta s})^T;
\]

\[
f(x) = \begin{bmatrix}
\frac{-x_1}{T_r} - p \cdot x_2 \cdot x_3 \\
\frac{-x_2}{T_r} + p \cdot x_1 \cdot x_3 \\
\frac{-J}{T_L} x_3
\end{bmatrix};
\]

\[
g(x) = \begin{bmatrix}
g_1(x) \\
g_2(x)
\end{bmatrix} = \begin{bmatrix}
\frac{L_m}{T_r} & 0 \\
0 & \frac{L_m}{T_r} \\
\frac{-pL_m}{JL_r} x_2 & \frac{pL_m}{JL_r} x_1
\end{bmatrix} \cdot u
\]

In order to linearize the system (1), two variables dependent on \(x\) only are considered as outputs of the system. They are taken here as:

\[
\phi_1 (x) = z_1 = x_1^2 + x_2^2 = \Phi_{\alpha r}^2 + \Phi_{\beta r}^2
\]

\[
\phi_2 (x) = z_2 = x_3 = \Omega
\]

The relative degrees \(r_i\) (\(i = 1, 2\)) for each output \((z_1, z_2)\) is in this case, \(r_1 = 1\) and \(r_2 = 1\).

This implies that the full-linearization is not realized, so, another variable \(\phi_3 (x)\), which represents the internal dynamic must be added. In this work, we choose the following one [1]:

\[
\phi_3 (x) = z_3 = \text{atan}(x_2/x_1) + k \pi \text{ with } k = \begin{cases}
0 & \text{if } x_1 > 0 \\
1 & \text{if } x_1 < 0
\end{cases}
\]

Consequently, the canonical form of the system is given by:

\[
\begin{align*}
\dot{z}_1 &= \frac{2}{T_r} z_1 + \frac{2L_m}{T_r} x_1 - \frac{2L_m}{T_r} x_2 \cdot u_1 \\
\dot{z}_2 &= -\frac{pL_m}{JL_r} x_2 + \frac{pL_m}{JL_r} x_1 \cdot u_2 \quad (4a)
\end{align*}
\]

\[
\dot{z}_3 = p \cdot z_2 + \frac{L_m}{T_r} \left( \frac{x_1 u_2 - x_3 u_1}{z_1} \right) 
\]

The system (4a) can be presented in the following matrix form:
\[ \dot{z}_r = A(z) + B(z) \cdot u \]  

(5)

where: \( z_r = (z_1, z_2)^T \), \( u = (u_1, u_2)^T \) and \( z = (z_1, z_2, z_3)^T \)

\( B(z) \): representing the decoupling matrix.

The linearizing feedback is defined as follows:

\[
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} = \frac{1}{\det(B(z))} \begin{bmatrix}
    \frac{pL_m}{JL_r} x_1 - \frac{2L_m}{T_r} x_2 \\
    \frac{pL_m}{JL_r} x_2 - \frac{2L_m}{T_r} x_1
\end{bmatrix} \begin{bmatrix}
    \dot{z}_1 + \frac{2}{T_r} z_1 \\
    \dot{z}_2 + \frac{T_L}{J}
\end{bmatrix}
\]

(6)

However, we notice in this case that system (6) is rather complex, and depends closely on motor parameters, state variables, and external perturbations. That makes its representation by an approximate model not obvious.

In order to minimize the number of input variables and reduce the dependence of the system (6) on parameter variations and external perturbations, while maintaining decoupling between the two subsystems ordered by the command variables \( v_1 \) and \( v_2 \), we propose a new reformulation of the system expressed as \([19–22]\):

\[
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} = \frac{1}{\det(B(z))} \begin{bmatrix}
    x_1 - x_2 \\
    x_2 - x_1
\end{bmatrix} \begin{bmatrix}
    \frac{pL_m}{JL_r} \\
    \frac{2L_m}{T_r}
\end{bmatrix} \begin{bmatrix}
    \dot{z}_1 + \frac{2}{T_r} z_1 \\
    \dot{z}_2 + \frac{T_L}{J}
\end{bmatrix}
\]

(7)

By considering \( v_1 \) and \( v_2 \) as the new command variables, \( u_1 \) and \( u_2 \) are given as follows:

\[
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} = \begin{bmatrix}
    x_1 - x_2 \\
    x_2 - x_1
\end{bmatrix} \begin{bmatrix}
    v_1 \\
    v_2
\end{bmatrix}
\]

(8)

The resulting system governed by the above state and input transformations is given by:

\[
\begin{align*}
\dot{z}_1 &= f_1(z) + g_1(z) \cdot u_1 = -\frac{2}{T_r} z_1 + \frac{2L_m}{T_r} z_1 \cdot v_1 \\
\dot{z}_2 &= f_2(z) + g_2(z) \cdot u_2 = -\frac{T_L}{J} + \frac{pL_m}{JL_r} z_1 \cdot v_2
\end{align*}
\]

(9)

The system is made up of two subsystems, each one is put in canonical form and dependent on one command \( v_i \). The block diagram of the resulting nonlinear feedback control system (9) is depicted in Figure 1.

![Block diagram of the nonlinear feedback based control system](image-url)
The block describing the relation between \( u \) and \( v \) cannot be replaced by an approximate system, because of no uncertainty between internal and external command variables is tolerated. What means that good observation of flux components in the stator frame is necessary [9, 23], leading to adequate variable command \((u_1, u_2)\) and good control performance.

3. INTRODUCTION OF THE SLIDING MODE CONTROL (SMC)

The basic principle of sliding mode control consists in moving the state trajectory of the system towards a predetermined surface called the sliding or switching surface and in maintaining it around this latter with an appropriate switching logic [1, 3, 5].

The two-time-scale behavior of a singularly perturbed system is characterized by a slow and a fast motion in the system dynamics. The slow motion, approximated by a reduced model, is usually related to a variable structure system as a sliding mode. It concerns the attractivity of the state trajectory to the sliding surface. The fast transients, represented by a boundary layer correction portion, correspond to the reaching mode before the state trajectory lies on the sliding surface. This latter is very important in terms of application of nonlinear control techniques, because it eliminates the uncertain effect of the model and the external perturbation. Among the strategies of the sliding mode control available in the literature, one can choose for the controller characterised by the following control law [1, 3, 5]:

\[
U = U_{eq} + U_n
\]  

(10)

Where \( U_{eq} \): equivalent control,

\( U_n \): robust control.

3.1. Equivalent Control \( U_{eq} \)

This control is selected so that the system exhibits desirable dynamical behavior when its trajectories are confined to the sliding surface. A necessary condition for the system state trajectory to remain on the sliding surface is \( \dot{S}(\cdot) = 0 \) which corresponds to the equivalent control applied for the nominal system. In the present work, four equivalent controls \( U_{eq} \) are adopted for our cascade structure of the motor and are calculated by imposing \( \dot{S}_i(z) = 0 \) and \( \dot{S}_j(u) = 0 \), where \( i : 1, 2 \) and \( j : 3, 4 \) (Figure 1) [19].

3.2. Robust Control \( U_n \)

If the initial state is not on the sliding surface, or there is a displacement of the representative point from this latter due to parameter variations and/or disturbances, the controller must be conceived in a way that it can drive the system state variable to the sliding mode \( S(\cdot) = 0 \). The phase where the system trajectory guarantees that the state moves towards and reaches the sliding surface is called the reaching mode or reaching phase ensured by a control variable called “robust control or switching control” which is designed by \( U_n \) in Equation (10) and developed to guarantee the reaching condition \( \dot{V}(\cdot) = S^T \cdot \dot{S} \) in the presence of parameter uncertainties and disturbance uncertainties [1, 5].

In a conventional variable structure control the reachability control generates a high control activity as it depends on the magnitude \( G(\cdot) \). Since it was first taken as constant, a relay function is very harmful to the actuators and may excite the unmodeled dynamics of the system. This is known as a chattering phenomenon [1, 5, 6, 24]. The main cause of the chattering and the large control energy is the use of a control law that depends only on the known upper bounds of uncertainties and disturbances. Ideally, to reach the sliding surface, the chattering phenomenon should be eliminated. However, in practice, chattering can only be reduced.

During the last years, the reduction of chattering became a focus of much research [1, 5, 6–9, 19–22, 24]. Among these, the first approach to reduce chattering was to introduce a boundary layer around the sliding surface and to use smooth functions to replace the discontinuous part of the control action. In this work the following function, which gives higher performances as it uses an exponential function for smoothing, is proposed by the authors in [19–22] (Figure 2):

\[
G(S) = \begin{cases} 
K - (K - k) \cdot \exp \left(- \frac{\|S(\cdot)\| \varepsilon}{\sigma} \right) & ; \quad \|S(\cdot)\| > \varepsilon \\
\frac{k}{\varepsilon} & ; \quad \|S(\cdot)\| \leq \varepsilon
\end{cases}
\]  

(11)
The constant $K$ is linked to the speed of convergence towards the sliding surface of the process (the reaching mode). $k$ is the minimal value of $G(S)$, necessary to compensate uncertainties and disturbances to guarantee convergence to the boundary layer.

The value of $\varepsilon$ is important as it affects simultaneously the switching frequency and the tracking of the sliding surface [19–22].

In this contribution, the sliding mode controllers are used as shown in Figure 3. The average behavior of the PWM controlled inverter is obtained by using the system model of an induction motor to generate the discrete voltages inputs ($V_{as}^*, V_{bs}^*, V_{cs}^*$) [19–22].

### 3.3. Design of the Switching Surfaces

In this work, four sliding surfaces are taken as [19]:

\[
\begin{align*}
S_1(z_1) &= e_1(z_1) = \Phi_{ref}^2 - \Phi_2^2 = z_{1ref} - z_1 \\
S_2(z_2) &= e_2(z_2) = \Omega_{ref} - \Omega = z_{2ref} - z_2 \\
S_3(u_1) &= e_3(u_1) = i_{as}^* - i_{as} \\
S_4(u_2) &= e_4(u_2) = i_{ps}^* - i_{ps}
\end{align*}
\]

(12)

with : $\Phi_{ref}$ and $\Omega_{ref}$, being respectively, the reference values of the flux and the speed.

$S_1(z_1)$ and $S_2(z_2)$ are related to the outer loop, whereas $S_3(u_1)$ and $S_4(u_2)$ are related to the inner loop. The $i_{as}^*$ and $i_{ps}^*$
references are determined by the outer loop, and take respectively the values of $i_{\alpha}$ and $i_{\beta}$.

### 3.3. Development of the Control Laws

By using (9) and (10), the two regulators' control laws, for the flux and the speed, of the external loop are given by the following equations:

#### 3.3.1. For the Flux and the Speed Regulators


deq \begin{align*} 
    v_1 & = \left( -\lambda_1 \cdot \dot{z}_{1\text{ref}} + \lambda_1 \cdot \frac{2}{T_r} z_1 \right) + \left( \frac{-1}{2L_m} \frac{z_1}{T_r} \right) \cdot \hat{S}_{1d} = v_{1\text{eq}}(z_1) + b_1(z_1) \cdot \hat{S}_{1d} \\
    v_2 & = \left( -\lambda_2 \cdot \dot{z}_{2\text{ref}} + \lambda_2 \cdot \frac{C_r}{J} \right) + \left( \frac{-1}{pL_m} \frac{z_1}{J L_r} \right) \cdot \hat{S}_{2d} = v_{2\text{eq}}(z_1) + b_2(z_1) \cdot \hat{S}_{2d} 
\end{align*}

with:

\begin{align*} 
    \hat{S}_{1d}(S_1) & = M_1(S_1) \cdot \text{sgn}(S_1) \\
    \hat{S}_{2d}(S_2) & = M_2(S_2) \cdot \text{sgn}(S_2) \\
    M_i & = K_i - (K_i - k_i) \cdot \exp \left( -\frac{|S_i(z_i)| - \varepsilon_i}{\sigma_i} \right) ; \quad |S_i(z_i)| > \varepsilon_i , \quad i = 1, 2
\end{align*}

In the same way, the two regulators' control laws, for the control variables $i_{\alpha}$ and $i_{\beta}$, of the internal loop, are given as follows:

#### 3.3.2. For the Control Variable $i_{\alpha}$


deq \begin{align*} 
    S(i_{\alpha}) \cdot \hat{S}(i_{\alpha}) < 0 \Rightarrow V_{\alpha} & = V_{\alpha\text{eq}} + V_{\alpha\text{n}} \\
    V_{\alpha\text{eq}} & = \sigma \cdot L_s \frac{d}{dt} i_{\alpha} + R_s i_{\alpha} - \frac{L_m}{T_r} \omega_r \Phi_r \\
    V_{\alpha\text{n}} & = -\left( K_3 - (K_3 - k_3) \cdot \exp \left( -\frac{|S_3(u)| - \varepsilon_3}{\sigma} \right) \right) \cdot \text{sgn} (S_3(u)) \quad ; \quad |S_3(u)| > \varepsilon_3
\end{align*}

#### 3.3.3. For the Control Variable $i_{\beta}$


deq \begin{align*} 
    S(i_{\beta}) \cdot \hat{S}(i_{\beta}) < 0 \Rightarrow V_{\beta} & = V_{\beta\text{eq}} + V_{\beta\text{n}} \\
    V_{\beta\text{eq}} & = \sigma \cdot L_s \frac{d}{dt} i_{\beta} + R_s i_{\beta} + \frac{L_m}{T_r} \omega_r \Phi_r \\
    V_{\beta\text{n}} & = -\left( K_4 - (K_4 - k_4) \cdot \exp \left( -\frac{|S_4(u)| - \varepsilon_4}{\sigma} \right) \right) \cdot \text{sgn} (S_4(u)) \quad ; \quad |S_4(u)| > \varepsilon_4
\end{align*}
To satisfy the stability condition of the system, the gains $K_1$, $K_2$, $K_3$, and $K_4$ should first be taken as positive and then adjusted to the appropriate values which correspond to the highest performances of the system [19–22].

$K_1$ and $K_4$ take the acceptable values of the transient stator voltage in the direct and in the quadrature axis respectively, then:

$K_3 = V_{\alpha \text{max}}$ and $K_4 = V_{\beta \text{max}}$

Using the Park transformation, the reference voltages in the ($a$, $b$, $c$) coordinates are given by:

$$\begin{bmatrix}
V_{a} \\
V_{b} \\
V_{c}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & 0 & \sqrt{3}/2 \\
-1/2 & \sqrt{3}/2 & -1/2 \\
-1/2 & -\sqrt{3}/2 & -1/2
\end{bmatrix} \begin{bmatrix}
V_{\alpha} \\
V_{\beta}
\end{bmatrix}$$

(20)

In Equation (1) the effect of load torque on the stator current control ($i_{\beta s}$) can be noticed. Since the load torque is not exactly known at any time, its estimation is necessary in order to achieve the high possible performance. On the other hand, by using this estimation, one can check the robustness of the sliding mode controller in terms of load torque variations.

This estimation is achieved by measuring the quadrature stator current, the speed, and its derivative. The mechanical equation gives:

$$T_L = \frac{3}{2} \frac{pL_m}{L_r} (x_3x_2 - x_1x_4) - J \frac{d\Omega}{dt} - f_r \cdot \Omega$$

(21)

4. VALIDATION OF THE CASCADE SLIDING MODE CONTROLLER (SMC)

The first test concerns a no-load starting of the motor with a reference speed $\Omega_{ref} = 100$ rd/sec. A load torque ($T_L = 10$ Nm) is applied then between $t = 0.5$ sec and $t = 1$ sec, which is followed at $t = 1.5$ sec by a reverse of speed from 100 rd/s to –100 rd/s.

The test results obtained are shown in Figure 4.

Figure 4. Simulation results for a cascade structure using SMC controllers
The waveforms depicted in the above figure show that the ideal variable decoupling is established, despite the load variations. Owing to the constant flux control, a quick speed response is thus obtained. Besides, this speed response is very close to the desired reference.

It is clearly shown that during a load torque perturbation and the reverse of speed operation, the actual rotor speed tracks the desired speed after a small transient state.

The step changes in the load torque and the speed response cause step changes in the torque response without any effects on the rotor flux components responses ($\Phi_\alpha$, $\Phi_\beta$), which are maintained constants, due to the decoupled control system between speed and rotor flux. A cascade structure with sliding mode control has been simulated using the motor parameters given in Appendix B. Thus, the speed regulation is obtained using such a controller in spite of the presence of severe disturbances such as load torque step changing and speed reverse.

Concerning the chattering phenomenon, this latter appears in the torque response due to the discontinuous characteristic of the controller. It cannot be eliminated completely.

5. FUZZY SLIDING MODE CONTROL OF AN INDUCTION MOTOR

In the previous section, it was shown that the variable structure control maintains robustness and improves the performances of the system. However, the phenomenon of chattering represents its main defect. Further, the practical implementation of the controller as a whole could be difficult. To overcome these problems and improve the performances a combination of fuzzy logic control and sliding mode control were proposed [7–9]. In this contribution, a new combination of fuzzy logic with sliding mode control proposed to an induction motor drive.

5.1. Algorithm for the Proposed Fuzzy Sliding Mode Control

Step 1: Construction of a Fuzzy Partition for the Input Variables

The algorithm starts with the construction of the fuzzy partition of the universe of discourse $\tilde{S}_{1}(\tilde{e}_1)$ for the component $z_1$ [7–9, 19]. $M_1$ intervals $I_{z_1}^{k_1}$ are defined and for each one a membership function is associated. Linguistic labels are given to the resulting fuzzy sets $F_{z_1}^{k_1}; k_1 = 1, \ldots, M_1$.

The universe of discourse of the linguistic variable $z_1$ verify:

$$U_{z_1} = \bigcup_{k_1=1}^{M_1} I_{z_1}^{k_1}$$  \hspace{1cm} (22)

In this case, a triangular membership function is associated to each interval $I_{z_1}^{k_1}$ as shown in Figure 5.

![Figure 5. Fuzzy partition of the input variable $z_1$](image)

The fuzzy sets $F_{z_1}^{k_1}$ form a fuzzy partition $P_F(U_{z_1})$ of the universe of discourse $U_{z_1}$, defined as:

$$P_F(U_{z_1}) = \left\{ F_{z_1}^{k_1} \mid k_1 = 1, \ldots, M_1 \right\}$$  \hspace{1cm} (23)

Therefore, the fuzzy state variable $\tilde{z}_1$ represents one of the $M_1$ fuzzy sets $F_{z_1}^{k_1}$ in the universe of discourse $U_{z_1}$. 
The width of the support, the cardinality of the fuzzy partition, and the shape of the membership functions are chosen by the designer.

**Step 2: Construction of the Fuzzy Sets for the Functions \( u_{1eq}, u_{1eq}, b_1, \) and \( b_2 \)**

This step concerns the construction of the fuzzy functions \( \tilde{u}_{1eq}(\tilde{z}_1), \tilde{u}_{2eq}(\tilde{z}_1), \tilde{b}_1(\tilde{z}_1), \) and \( \tilde{b}_2(\tilde{z}_1), \) given the practical range of \( \eta(t). \) First the function \( u_{1eq}(\cdot) \) is considered.

In the proposed modeling approach, there is a need to evaluate the range of values \( I_{f_i}^{(k)} \) of the function \( f_i(z_1), \) which is defined as:

\[
I_{u_{1eq}}^{(k)} = \left\{ u_{1eq} \left( z_1(t), \eta(t) \right) \mid z_1(t) \in I_{z_1}^{k_1}, \ \eta(t) \in P \right\}
\]  

(24)

where: \( P \) represents the range of variations of all the parameters of the machine and the external perturbations.

The universe of discourse \( U_{u_{1eq}} \) of \( u_{1eq}(z_1(t),\eta(t)) \) is defined by:

\[
U_{u_{1eq}} = \bigcup_{k_1=1}^{M_1} I_{u_{1eq}}^{(k_1)}
\]

(25)

In the same way, as for the function \( u_{1eq}(\cdot) \), the intervals and the range of values of the scalar functions \( u_{2eq}(\cdot), b_1(\cdot) \) and \( b_2(\cdot) \) are determined. This gives:

\[
I_{u_{2eq}}^{(k_1)} = \left\{ u_{2eq} \left( z_1(t), \eta(t) \right) \mid z_1(t) \in I_{z_1}^{k_1}, \ \eta(t) \in P \right\}
\]

(26)

\[
U_{u_{2eq}} = \bigcup_{k_1=1}^{M_1} I_{u_{2eq}}^{(k_1)}
\]

(27)

and

\[
I_{b_1}^{(k_1)} = \left\{ b_1 \left( z_1(t), \eta(t) \right) \mid z_1(t) \in I_{z_1}^{k_1}, \ \eta(t) \in P \right\}
\]

(28)

\[
U_{b_1} = \bigcup_{k_1=1}^{M_1} I_{b_1}^{(k_1)}
\]

(29)

\[
I_{b_2}^{(k_1)} = \left\{ b_2 \left( z_1(t), \eta(t) \right) \mid z_1(t) \in I_{z_1}^{k_1}, \ \eta(t) \in P \right\}
\]

(30)

\[
U_{b_2} = \bigcup_{k_1=1}^{M_1} I_{b_2}^{(k_1)}
\]

(31)

For: \( k_1 = 1, \ldots, M_1. \)

**Step 3: Definition of the Fuzzy Sets for the Functions**

Now, a fuzzy partition of \( U_{u_{1eq}} \) is constructed by covering the intervals \( I_{u_{1eq}}^{(k_1)} \) with appropriate membership functions \( \mu_{u_{1eq}}^{(k_1)}. \) Thus, the fuzzy partition of \( U_{u_{1eq}}, P_F\{U_{u_{1eq}}\}, \) is given by:

\[
P_F(U_{u_{1eq}}) = \left\{ F_i^{(k_1)} \mid k_1 = 1, \ldots, M_1 \right\}
\]

(32)

The form of the membership function is chosen by the designer. In the present case, Figure 6 shows the membership function used.
By proceeding in the same manner for the two other functions $g_1(z_1)$ and $g_2(z_1)$, the following partitions are obtained:

$$P_F(U_{u_{1\text{eq}}}) = \left\{ F_{u_{1\text{eq}}}^{(k_1)} \mid k_1 = 1, \ldots, M_1 \right\}$$  \hspace{1cm} (33a) $$P_F(U_{b_1}) = \left\{ F_{b_1}^{(k_1)} \mid k_1 = 1, \ldots, M_1 \right\}$$  \hspace{1cm} (33b) $$P_F(U_{b_2}) = \left\{ F_{b_2}^{(k_1)} \mid k_1 = 1, \ldots, M_1 \right\}$$  \hspace{1cm} (33c)

In the same way, the fuzzy partitions of $u_{1\text{eq}}(\cdot)$, $b_1(\cdot)$ and $b_2(\cdot)$ are realized.

**Step 4: Rule Base Deduction**

The fuzzy functions $\widetilde{u}_{1\text{eq}}$, $\widetilde{u}_{2\text{eq}}$, $\widetilde{b}_1$, and $\widetilde{b}_2$ can be defined respectively as:

$$\widetilde{u}_{1\text{eq}} : \quad P_F(U_{z_1}) \rightarrow \quad \widetilde{z}_1 = F_{z_1}^{k_1} \rightarrow$$  \hspace{1cm} (34) $$\text{where:} \quad \widetilde{u}_{1\text{eq}}(\widetilde{z}_1) = F_{u_{1\text{eq}}}^{(k_1)} \in P_F(U_{u_{1\text{eq}}}) \text{ for } k_1 = 1, \ldots, M_1.$$ 

and

$$\widetilde{u}_{2\text{eq}} : \quad P_F(U_{z_1}) \rightarrow \quad \widetilde{z}_1 = F_{z_1}^{k_1} \rightarrow$$  \hspace{1cm} (35) $$\text{where:} \quad \widetilde{u}_{2\text{eq}}(\widetilde{z}_1) = F_{u_{2\text{eq}}}^{(k_1)} \in P_F(U_{u_{2\text{eq}}}) \text{ for } k_1 = 1, \ldots, M_1.$$ 

and

$$\widetilde{b}_1 : \quad P_F(U_{z_1}) \rightarrow \quad \widetilde{z}_1 = F_{z_1}^{k_1} \rightarrow$$  \hspace{1cm} (36) $$\text{where:} \quad \widetilde{b}_1(\widetilde{z}_1) = F_{b_1}^{(k_1)} \in P_F(U_{b_1}) \text{ for } k_1 = 1, \ldots, M_1, \text{ and } i = 1, 2, 3 \text{ being the set of the real numbers.}$$

Using “If–Then” rules, the representation of the fuzzy functions $\widetilde{u}_{1\text{eq}}$, $\widetilde{u}_{2\text{eq}}$, $\widetilde{b}_1$, and $\widetilde{b}_2$ become concrete by using $M_1$ fuzzy sets as: 

\[\]
\( R^{(k_i)} : \)

\[
\begin{align*}
\text{IF} & \quad z_1 \text{ is } F^{k_i}_{z_1} \\
\text{THEN} & \quad \left[ u_{1eq}(z_1, \eta), u_{2eq}(z_1, \eta), b_1(z_1, \eta), b_2(z_1, \eta) \right] \text{ are } \left[ F^{(k_i)}_{u_{1eq}}, F^{(k_i)}_{u_{2eq}}, F^{(k_i)}_{b_1}, F^{(k_i)}_{b_2} \right].
\end{align*}
\]

(37)

**Step 5. Construction of the Fuzzy Switching Functions**

\( U_{s_i} \subset R \) denotes the set of all possible values that can take the switching function \( S_i(z) \) in (12); \( i = 1, 2 \). The set \( U_{s_i} \) is then defined as:

\[
U_{s_i} = \left\{ S(z_i) \in 3 \mid e_i \in U_{e_i} \right\}; \quad i = 1, 2
\]

(38)

A fuzzy switching function is defined by:

\[
\tilde{S}_i : \quad P_F(U_{e_i}) \quad \rightarrow \quad \vec{e}_i = F^{k_i}_{e_i}
\]

(39)

where, by using (13) and (14):

\[
\tilde{S}_1(\vec{e}_1) = \vec{e}_1 \quad \text{and} \quad \tilde{S}_2(\vec{e}_2) = \vec{e}_2
\]

In this step, the fuzzy partition \( P_F(U_{s_i}) \) for the range of values \( U_{s_i} \) of the switching functions \( S_1(z_1) \) and \( S_2(z_2) \) is constructed as:

\[
P_F(U_{s_i}) = \left\{ F^{k_i}_{s_i} = F^{k_i}_{e_i} \mid k_i = 1, \ldots, M_i \right\}
\]

(40)

Hence, the fuzzy switching function \( \tilde{S}_i(\vec{e}_i) \) \((i = 1, 2)\) represents the \( M_i \) fuzzy sets defined above.

**Step 6. Choice of the Desired Fuzzy Dynamics**

The phase during which the fuzzy state trajectories are driven towards the fuzzy sliding surface is referred to as the fuzzy reaching mode. The control of the dynamics of the fuzzy system during this mode may be made possible by specifying the dynamics of the fuzzy switching functions \( \tilde{S}_{d_i} \) \((i = 1, 2)\).

For each fuzzy set \( F^{k_i}_{s_i} \) \((i = 1, 2)\), the following dynamics, that characterize the fuzzy switching function \( \tilde{S}_{d_i} \), which is related to the nonlinear functions \( G_1(S_1) \) and \( G_2(S_2) \), are proposed:

\[
F^{k_i}_{d_j} = -\omega_j \sim F^{k_i}_{M_j(PF_{s_j})} + (\omega_j - 1) \sim F^{k_i}_{M_j(NF_{s_j})}; \quad i = 1, 2
\]

(41)

with:

\[
\omega_j = \begin{cases} 
1 & \text{if } S_j \in \text{supp}(F^{k_i}_{s_j}) \cap U_{s_j}^+ \\
0 & \text{if } S_j \in \text{supp}(F^{k_i}_{s_j}) \cap U_{s_j}^-
\end{cases}
\]

\text{supp}(F^{k_i}_{s_j}) \text{ is the support of the fuzzy set } F^{k_i}_{s_j}.

\( U_{s_i}^+ \subset 3^+ \) and \( U_{s_i}^- \subset 3^- \) are respectively the positive and the negative part of the universe of discourse \( U_{s_i} \).
$PF_{x_i}$ and $NF_{x_i}$ are respectively the positive and the negative part of the fuzzy set $F_{x_i}$.

$F_{G(PF_{x_i})}^{k_i}$ the fuzzy sets that can take the values of $G_i(S_i)$ if $S_i = PF_{x_i}^{k_i}$.

$F_{G(NF_{x_i})}^{k_i}$ the fuzzy sets that can take the values of $G_i(S_i)$ if $S_i = NF_{x_i}^{k_i}$.

The rule base that characterizes the desired fuzzy dynamic, consists of the rules:

$$\mathcal{R}^{(k_i)} : \text{IF } \bar{z}_i \text{ is } F_{\bar{z}_i}^{k_i} \text{ THEN } \bar{S}_t(\bar{e}_i) \text{ is } F_{\bar{S}_t}^{k_i}$$

Prior to extending the conventional sliding mode control to fuzzy sliding mode control algorithm, the following fuzzy reaching condition must be met which is defined by:

$$\exists \bar{N} \ni \bar{S}(\bar{e}) \cdot \bar{S}(\bar{e}) \subset \bar{N}$$

$\bar{N}$ is a fuzzy set whose support is strictly negative.

The fuzzy reaching condition (43) is equivalent to the condition (12) such as:

$$\forall \bar{S}(\bar{e}) \in \text{Supp} \left( \bar{S}_t(\bar{e}) \right) \setminus \{0\} \text{ and } \forall \bar{S}_t(\bar{e}) \in \text{Supp} \left( \bar{S}_t(\bar{e}) \right) \setminus \{0\}$$

**Step 7. Design of the Fuzzy Control Law**

Given the dynamics of the fuzzy reaching mode and the Equations (13) and (14), the resulting fuzzy control laws are determined as follows [7–9, 19–22]:

$$\bar{v}_1 = \bar{v}_{1eq}(\bar{z}_1) + \tilde{b}_1(\bar{z}_1) \sim \bar{S}_{1d}(\bar{z}_1)$$

$$\bar{v}_2 = \bar{v}_{2eq}(\bar{z}_2) + \tilde{b}_2(\bar{z}_2) \sim \bar{S}_{2d}(\bar{z}_2)$$

The fuzzy sets $F_{v_1}^{(k_1,k_2)}$ and $F_{v_2}^{(k_1,k_2)}$ are deduced from the above equations:

$$F_{v_1}^{(k_1)} = F_{v_{1eq}}^{(k_1)} \sim F_{b_1}^{(k_1)} \sim F_{s_{d1}}^{(k_1)}$$

$$F_{v_2}^{(k_1,k_2)} = F_{v_{2eq}}^{(k_1)} + F_{b_2}^{(k_1)} \sim F_{s_{d2}}^{(k_1)}$$

The fuzzy control rule base consists of the following $M_1$ fuzzy control rules:

$$\mathcal{R}^{(k_1)}_{v_1} : \text{IF } (z_1, e_1)^t \text{ is } (F_{z_1}^{k_1}, F_{e_1}^{k_1})^t \text{ THEN } v_1 \text{ is } F_{v_1}^{(k_1)}$$

$$\mathcal{R}^{(k_1,k_2)}_{v_2} : \text{IF } (z_1, e_2)^t \text{ is } (F_{z_1}^{k_1}, F_{e_2}^{k_2})^t \text{ THEN } v_2 \text{ is } F_{v_2}^{(k_1,k_2)}$$
5.2. Defuzzification of the Fuzzy Control Laws

Since the control to be fed back to the plant has to be crisp, a defuzzification stage has to be included in the controller. The following weighted average defuzzification method is used in this case:

\[
\sum_{k_{1}} \sum_{k_{2}} \frac{v_{f}}{\sum_{k_{1}} \sum_{k_{2}} \min \left(\mu_{k_{1}}, \mu_{k_{2}}\right) \cdot S_{v_{f}}^{(k_{1}, k_{2})} \cdot S_{v_{f}}^{(k_{1}, k_{2})}}
\]

where:

\[v_{f}^{(k_{1}, k_{2})}\]

is the control input value that achieves a maximum membership degree with respect to the fuzzy set \(v_{f}^{(k_{1}, k_{2})}\), i.e.:

\[
v_{f}^{(k_{1}, k_{2})} = \sup_{v_{f} \in U_{v_{f}}} \mu_{v_{f}}^{(k_{1}, k_{2})} (v_{f}) \quad ; \quad i = 1, 2
\]

6. VALIDATION OF THE SPEED REGULATION WITH FUZZY SLIDING MODE CONTROL

A simulation of linguistic fuzzy sliding mode control (FSMC) of an induction motor is carried out. In order to be able to compare the results, the same tests as those for the conventional sliding mode control (SMC) were carried out for different numbers of partitions (3, 7, and 15).

The comparison between these two system controls is illustrated in Figure 7. The uncertainties are not considered in fuzzy control development. The results confirm that the approximation is more precise if the number of input space partition increases (the approximation is better from a partitioning in 7 fuzzy sets).

To show the effects of this precision on the system behavior, we carried out a simulation relating to each partition, while keeping the same test conditions. The results obtained are presented in Figure 8. The waveforms look like those obtained with SMC. We notice, in this case too, that the results demonstrate the fact that the thinner the fuzzy partition of the input variables, the better the performance the fuzzy controller.

The coefficients in Equations (9) are all dependent on the motor parameters. Since, these parameters may vary during on-line operation due to temperature or saturation effects. So, it is important to investigate the sensitivity of the complete system, using the proposed fuzzy robust controllers, to parameters changes. In the same way that for SMC, a test of robustness was carried out for 3 and 5 partitions, for a variation of the rotor resistance from 50% up to 150% compared to nominal resistance, and for unestimated load torque. The results obtained are depicted in Figure 9. The waveforms are similar to those of Figure 5, by keeping a good robustness. These results also show that FVSC with a partitioning of 3 fuzzy sets of the universe of discourse of the input variables is robust, in spite of the bad approximation of the original controller. That proves that the model suggested in its canonical form is effective, and that the FVSC take well into account the range of uncertainties.

7. CONCLUSION

In this paper, a fuzzy sliding mode control approach for induction motor drive has been developed and presented. For this, a new decoupled model of the motor is first proposed. Then, a set of simple surfaces and associated control laws have been synthesized. For the development of the attractivity control law, a smooth control function with a threshold has been adopted to reduce the chattering phenomenon. However, the magnitude of this control law depends closely on the upper bound of uncertainties, which include parameter variations and external disturbances. This bound is difficult to obtain prior to motor operation. Even though, the piecewise smooth function \(G(S)\) improves the performance control and leads to a relative reduction of the chattering phenomenon, its practical implementation could be difficult.

To solve the above problems, a new decoupled model, taking into account all the uncertainties and perturbations, without imposing any structural conditions on these latter, is used for the development of the fuzzy sliding mode controller. Notice that only the ranges of uncertainties are required in this step and not the exact value of each parameter.
Figure 7. Comparison between conventional and fuzzy model based control systems

(a) Errors depending on $M_i$

(b) Error between conventional and fuzzy control systems for $M_i = 3$

(c) Error between conventional and fuzzy control systems for $M_i = 7$

(d) Error between conventional and fuzzy control systems for $M_i = 15$
The cascade structure associated with the proposed fuzzy sliding mode control is designed in order to improve the dynamical performances of the induction motor based drive and to keep protection of the overall system.

Simulation results reveal some very interesting features and show that the proposed fuzzy sliding mode control could be used as an alternative to the conventional sliding mode control of induction motors.

The implementation of the fuzzy representation $\tilde{G}(\tilde{S})$ is simpler in this case.

Also, the performance of the fuzzy sliding mode controllers, though encouraging in this example, should be tested in a higher order system, where the variable structure control would use the error and its derivatives.
Figure 9. Simulation results for different variations of \( r_r \)
(without load torque estimation and a variation of 5% of the inductances)
APPENDIX A: LIST OF PRINCIPLE SYMBOLS

\( i_{\alpha} \), \( i_{\beta} \): stator current \( \alpha-\beta \) axis references
\( v_{\alpha} \), \( v_{\beta} \): stator voltage \( \alpha-\beta \) axis references
\( i_{\alpha}, i_{\beta} \): stator current \( \alpha-\beta \) axis components
\( v_{\alpha}, v_{\beta} \): stator voltage \( \alpha-\beta \) axis components
\( \Phi_{\alpha}, \Phi_{\beta} \): rotor flux \( \alpha-\beta \) axis components
\( \Phi_r \): rotor flux command
\( R_s, R_r \): stator, rotor resistances
\( L_s, L_r \): stator, rotor inductances
\( L_m \): mutual inductance
\( T_s \): stator time constant \((L_s/R_s)\)
\( T_r \): rotor time constant \((L_r/R_r)\)
\( \sigma \): total leakage coefficient \((\sigma = 1 - L_m^2/(L_rL_s))\)
\( p \): pairs of poles
\( \Omega \): mechanical speed
\( J \): moment of inertia
\( f \): viscous friction coefficient
\( T_{em} \): torque command
\( T_L \): load torque

APPENDIX B: MACHINE PARAMETERS

The squirrel-cage induction motor of 1.5 Kw, 220 V, 2 poles, 1420 tr/min, 50 Hz.
\( R_s = 4.85 \, \Omega \); \( R_r = 3.805 \, \Omega \); \( L_s = 0.274 \, \text{H} \); \( L_r = 0.274 \, \text{H} \)
\( L_m = 0.258 \, \text{H} \); \( J = 0.031 \, \text{Kg.m}^2 \); \( f = 0.00114 \, \text{Nms} \)

REFERENCES


