ROBUST GOVERNOR DESIGN FOR HYDRO TURBINES USING A MULTIVARIABLE-CASCADE CONTROL APPROACH

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ABSTRACT

A robust governor design using a multivariable-cascade control approach is proposed for hydro turbine speed controls. The non-linear turbine model includes the effects of water hammer, travelling waves, and inelastic water penstocks. Disturbance and parameter uncertainties are briefly discussed to investigate the stability robustness and performance measures of the system. Bounds of the uncertainties in system parameters are required to design the robust governor. Polynomial $H_{\infty}$ control method is used in the design. The parameterized dynamic weighting functions of the design theory are selected to achieve the required control functions and robustness. The robust governor ensures that the overall system remains asymptotically stable for all bounded uncertainties and for system oscillations. Simulation results show the feasibility of the approach and the factors involved in the design such that the proposed robust governor improves the performance significantly even in the presence of the uncertainties in plant parameters.

Keywords: Robust governor, speed control, $H_{\infty}$ method, multivariable-cascade control.
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NOTATIONS
Polynomial notation is employed and the polynomials are assumed to be functions of the complex $s$ variable.

$R$  
Set of all real numbers.

$R(.)$  
Set of all real rational functions.

$R[.]$  
Set of finite polynomials with real coefficients.

$R_{mxm}(.)$  
Set of all real $(m \times m)$ matrices.

$R_{mxm}[.]$  
Set of polynomial $(m \times m)$ matrices.

(*)  
Complex conjugate transpose.

1. INTRODUCTION
Governors are used to regulate speeds of turbine–generators in power system primary units. Increasing number of interconnections, parallel operations of the turbine–generators in a power generation unit, higher transmission voltages and development of large power generating units have ensured that design of the governors remains a challenging and important problem [1–3]. Advanced control methods should be used in governor designs in order to realise full potential of the plant over a wide range of operating conditions [2]. Thus a governor has to manage the difference between the nominal plant model used in design and the true plant model. Bridging the gap between nominal model and true plant is the field of robust controller design [4–8].

The early power systems were small, often geographically remote from the consumer loads and individually isolated. Consequently, empirical and classical methods were used to adjust governor parameters using a single-input single-output (SISO) control structure [9,10]. Some supplementary signals were also used to improve the performance [3]. However, use of multiple measurements changed the control structure into multivariable [3,11]. But, the classical control theory that has been used for SISO control systems provides no systematic design methodology for design of multivariable controllers [12]. Consequently, conventional governors do not respond satisfactorily over whole range of plant operation [13]. Multivariable optimal control design methods were applied to power systems in [1970]s, and their advantages over the classical control methods have been demonstrated [4,10,14,15]. However, the selection methods for constant weighting matrices in these methods were heuristic [4,7], and the problem of measurement of all the state variables was a significant difficulty [16–18]. Gain scheduling control as set of gains corresponding to each point of desired operation has also been used [19], but has some certain disadvantages [20,21]. In recent years, there has been considerable interest in the application of robust control [4–8], adaptive control [3,15,22], and genetic algorithms [23] to design governors. However, there are still some practical problems in real-time application of the adaptive control to overcome for long-term operation of power systems. In [1980]s, Zames [24] introduced $H_{\infty}$ robust control in the presence of system uncertainties and disturbances. Desired robustness in the control systems has been achieved using dynamic weighting functions [5]. These functions were parameterized and linked with the controller and the closed-loop system dynamic behavior [8,21,25,26]. The $H_{\infty}$ design method also provides a systematic design algorithm, and has several advantages over the classical and conventional optimal design methods [5,8,21,25,26].

In this paper, a robust governor design using a multivariable-cascade control approach is proposed for hydro turbine speed controls. First-order minimum and non-minimum phase turbine models have been used so far in the governor designs [1,9,10,13], and these models are not sufficient to investigate the behavior of the plant [1,27]. The non-linear turbine model used in the present design includes the effects of water hammer, travelling waves, and inelastic water
penstocks. Polynomial $H_\infty$ control method is used in the robust governor design. The plant parameter uncertainties and disturbances are included in the design algorithms. The robust governor ensures that the overall system remains asymptotically stable even in the presence of bounded uncertainties in the plant parameters. Simulation results demonstrate that the robust governor improves the performance significantly.

2. SYSTEM DESCRIPTION

Multivariable-cascade governor diagram is illustrated in Figure 1. $\Delta \omega(s)$, $\Delta \delta(s)$, $\Delta G(s)$, and $\Delta P_m(s)$ are the incremental deviations of speed, load angle, gate position, and mechanical power, respectively. $\Delta \omega(s)$, $\Delta G(s)$, and $\Delta P_m(s)$ are used as supplementary signals. $U(s)$ is the control input signal and $U_d(s)$ is the manipulated input. $G_p(s)$ denotes pilot and gate servomotor model. Turbine and generator models are represented by $G_t(s)$ and $G_g(s)$, respectively. $G_a(s)$, $G_b(s)$, and $G_c(s)$ are the disturbance models for the input water, the output load, and the load angle, respectively. $G_p(s)$ denotes the integral of the speed. Disturbances $d_1, d_2, d_3, d_4$, and $d_5$ represent the water, turbine, load, speed, and load angle, respectively. $\Delta \omega_{ref}$ designates set-point for $\Delta \omega$. $C_m(s)$, $C_a(s)$, $C_b(s)$ and $C_c(s)$ are the governor controllers.

The control system given in Figure 1 consists of two different design procedures: (i) inner controller $C_{in}$ should be designed first; and (ii) simultaneous design of multivariable controllers $C_a(s)$, $C_b(s)$, and $C_c(s)$. It is assumed that the plant sub-models, $G_p(s)$, $G_t(s)$, $G_g(s)$, and $G_d(s)$ are free of unstable hidden modes. The disturbance signals $d_i$ are also assumed to be zero mean statistically independent stationary white driving-noise sources with finite variance such as $E[d_i(t)] = 0$. The covariances of these signals without loss of generality are taken to be equal to the unity for the worst-case conditions [5,8,21,25].

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2.1. Inner Gate Position Loop

Inner gate position loop can be considered as the inner loop of a typical cascade system [8,25,26] and is constructed for responding rapidly to changes in water head, flow, or pressure. Disturbance ($d_1$) is partially compensated before it affects the turbine speed. The closed loop gate position can be given as:

$$\Delta G = G_m^{-1}(G_a C_{in} U_d + G_d d_1)$$

where $G_m = [1 + G_p C_{in}]$.

2.2. Outer Multivariable Loop

Plant and disturbance models should be collected in matrix form for multivariable design [25]. The variables that are assumed to be measurable in the outer multivariable loop should be considered to develop a system matrix. Mechanical power can be given in terms of gate position and disturbance as:

![Figure 1. Multivariable-cascade governor control block diagram.](image-url)
\[
\Delta P_m = G_1 \Delta G + d_2 .
\]

Equation (1) is substituted into Equation (2) to obtain the mechanical power in more compact form as:

\[
\Delta P_m = G_1 G_4 U_a + G_4 G_4 G_4 d_1 + d_2 .
\]

Turbine speed is obtained using individual transfer functions, input, and disturbances as:

\[
\Delta \omega = G_1 G_4 G_4 U_a + G_4 \left( G_4 G_4 G_4 d_1 + d_2 \right) - G_1 G_4 d_3 + G_4 d_4 .
\]

Finally, load angle is obtained as:

\[
\Delta \delta = G_1 G_4 G_4 U_a + G_4 \left( G_4 G_4 G_4 d_1 + d_2 \right) - G_1 G_4 d_3 + G_4 d_4 + G_4 d_5 .
\]

The system can be represented in matrix form using Equation (3), Equation (4) and Equation (5) as:

\[
\Delta Y = W_p U_a + W_d d ,
\]

where \( \Delta Y \) is the output and \( d \) is the disturbance, \( \Delta Y = [\Delta P_m \Delta \omega \Delta \delta]^T, d = [d_1 d_2 d_3 d_4 d_5]^T \).

The true plant model \( W_p \) and disturbance model \( W_d \) in Equation (6) can be given in the matrix form:

\[
W_p = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix}, \quad W_d = \begin{bmatrix} G_{d1} & 0 & 0 & 0 \\ G_{d2} & G_{d2} & G_{d2} & 0 \\ G_{d3} & G_{d3} & G_{d3} \end{bmatrix}
\]

where \( G_j \) and \( G_{dj} \) are the real rational functions. The system open-loop model, Equation (6), should be re-arranged for the multivariable design as:

\[
\begin{bmatrix} \Delta P_m \\ \Delta \omega \\ \Delta \delta \end{bmatrix} = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix} \begin{bmatrix} U_a(s) + 0 G_2 & 0 & 0 \\ 0 G_2 & 0 & 0 \\ 0 & 0 G_4 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}
\]

where the disturbances \( d_1, d_2, d_3 \) are assumed to be zero-mean statistically independent white noise driving sources with unity-variance. New disturbance models, \( G_{d1}, G_{d2}, \) and \( G_{d3} \) can be obtained using the spectral factorization theorem [5,25]:

\[
G_d \phi_d G_d^* = \sum_{i=1}^{n} \sum_{j=1}^{n} G_{dj} \phi_{dj} G_{dj}^*
\]

where the disturbance covariances, \( \phi_d, \phi_{dj} \) are assumed to be unity for the worst case conditions [5].

### 2.3. Robustness and Stability

Stability robustness of the compensated system can be measured using unstructured additive and multiplicative uncertainty descriptions [26]. In practice, the multiplicative perturbation \( \Delta_m \) for a nominal plant \( W_n \) is usually bounded in magnitude as:

\[
|\Delta_m(j \omega)| \leq |\Delta_m(j \omega)| \quad \forall \omega \in R ,
\]

where \( \delta_m \) are some real rational functions. The true plant model can be represented in terms of the multiplicative uncertainty and nominal plant model as

\[
W_p = (I + \Delta_m) W_n ,
\]

Using Nyquist curve geometry the following is obtained in general for robust stability:

\[
\sup_{\omega \in \text{rad}} |\delta_m(j \omega) T_n(j \omega)| < 1 ,
\]

where \( T_n \) represents the nominal real rational complementary sensitivity function.
\[ T_s = W_s C_s (I + W_s C_s)^{-1} \in \mathbb{R}^{n_r}(s), \]  

where \( C_s \) denotes the controller, \( C_s \in \mathbb{R}^{n_r}(s) \), \( r \) and \( m \) denote number of outputs and inputs, respectively. Similarly, the additive uncertainty, \( \Delta \), may also be considered [26].

### 2.4. Disturbance and Load Rejection Criteria

The satisfactory elimination of the disturbances, \( d_i \), requires [8,21,25,26]:

\[
\sup_{\omega \in \Omega} \{ |\Delta \omega(j \omega)|d_i(j \omega) \}^1_{i=1,...,k} < 1
\]

(14)

where \( k \) is the number of disturbances. The perfect control system would keep the transfer functions \( \Delta \omega/d_i \), small in magnitude. Tyreus Load Rejection Criteria (TLC) can also be applied as reported in [26].

### 3. TURBINE MODEL

The important relationship in the speed control system is between the per-unit head, \( H_s \), at the turbine inlet and the water velocity, \( V \), in penstock [27]. Water hammer occurs due to the water velocity variations in the penstock. A pressure wave is propagated up to column because of the elasticity of the steel in the penstock and the compressibility of water. Thus the travel time of this wave becomes significant. Assuming a uniform conduit supplied from a large reservoir, the ratio of the incremental head \( (\Delta H_s) \) to the incremental water velocity \( (\Delta V) \) at the turbine inlet is given [27]:

\[
\frac{\Delta H_s(s)}{\Delta V(s)} = -\Omega_p - 0.5Z_p \tanh(T_s),
\]

(15)

where \( \Omega_p \) denotes friction factor of the penstock and \( T_s \) is the wave travel time. Normalized hydraulic surge impedance of the penstock, \( Z_p \), is a function of the water starting time \( (T_w) \) and the wave travel time \( (T_s) \), \( Z_p = T_u/T_s \). The water starting time is given as:

\[
T_w = \frac{L Q_{\text{base}}}{H_{\text{base}} A_p g},
\]

(16)

where \( L \) and \( A_p \) are the length and cross-sectional area of the penstock, \( H_{\text{base}} \) and \( Q_{\text{base}} \) are the per-unit base values of the water column head and the water flow rate, respectively, \( g \) is the acceleration due to gravity. The wave travel time depends on the length of the conduit and the water wave velocity \( (a) \) as \( T_s = L/a \). The friction factor is a function of the friction coefficient \( (k_p) \) and the steady-state water velocity \( (V_o) \), \( \Omega_p = 2k_p V_o \). The hyperbolic function in Equation (15) can be represented with the infinite product expansions:

\[
\tanh(T_s) = \frac{sT_s}{1 + \left( \frac{sT_s}{n \pi} \right)^2},
\]

(17)

The water velocity deviation \( (\Delta V) \) and the mechanical power deviation \( (\Delta P_m) \) are given [27]:

\[
\Delta V = 0.5 \Delta H_s + \Delta G
\]

(18)

\[
\Delta P_m = \Delta H_s + \Delta G.
\]

(19)

The turbine transfer function \( (\Delta G_i) \) in terms of the distributed parameter system, which includes water hammer effects, head loss due to friction, and effect of inelastic penstock, can be obtained by combining Equation (15), Equation (18), and Equation (19):

\[
\Delta G_i = \frac{1 - \Omega_p - Z_p \tanh(T_s)}{1 + 0.5 \Omega_p + 0.5 Z_p \tanh(T_s)},
\]

(20)

The generator model can be given as:
where $H$ is the inertia, $D$ is the generator damping and $\Delta P_{LOAD}$ is the load power deviation.

The plant pilot and servo motor sub-model ($G_p$) is given by

$$G_p = \frac{1}{(\tau_p s + 1)(\tau_s s + 1)},$$

(22)

where $\tau_p$, $\tau_s$ are the pilot and gate servo motor time constants. The range of the system parameters is:

$$pT \in \Gamma_{\Gamma}, \quad sT \in \Gamma_{\Gamma}, \quad wT \in \Gamma_{\Gamma}, \quad eT \in \Gamma_{\Gamma}, \quad H \in \Gamma_{\Gamma}, \quad D \in \Gamma_{\Gamma}.$$  

where $\Gamma_i$ and $\Gamma_i$ are lower and upper bounds of the parameters.

4. CASE STUDY

We consider data for robust design of a nominal hydroturbine system [1]. The range of the plant parameters for a typical turbine is chosen [9,10,13,14,27] to introduce uncertainty into the plant as:

$$pT \in [0.00, 50.02], \quad sT \in [0.05, 0.5], \quad wT \in [0.54], \quad H \in [2, 6.6], \quad D \in [01], \quad eT \in [0.2, 0.5].$$

The turbine model with the infinite product expansions may be approximated by a lumped parameter equivalent by retaining an appropriate number of terms of the expansions [27]. For most power system stability studies this approximation should be adequate for short- to medium-length penstocks [1,27]. Then, the third-order nominal turbine model is calculated for $n=1$ as:

$$G_3 = \begin{bmatrix}
0.28246 & 0.71265 & 15.6028 & 9.8413 \\
0.14123 & 0.71572 & 7.80143 & 9.8837 \\
\end{bmatrix}.$$  

It has been known that the inter-area oscillations affect the governing unit [1,8,27,28]. Then, the speed disturbance model ($G_s$) should include the inter-area modes up to 0.5 Hz., $G_s = l/(s + 2\pi f_o)$, where $f_o$ is the oscillating disturbance frequency. The load angle disturbance and water disturbance models, $d_G$ and $w_G$, may be assumed to be unknown and effective at all frequencies ($G_{dG} = 1$ and $G_{wG} = 1$). The unknown mechanical power disturbance, $d_z$, is caused by variation in turbine flow; hence the friction among rotational and fixed components and also friction by the water.

4.1. Inner Gate Position Loop Design

The controller, $C_u$, is designed first to regulate the deviations in gate position, which provides additional performance to the system. This loop may be considered as an inner-loop of a simple cascade control [25,26] and the design rules of a cascade control system are valid. As a requirement, the designed inner closed gate position loop time constant should be at least three times smaller than the outer open-loop plant dominant time constant. Polynomial SISO $H_\infty$ control design theory [5,8,26] can be considered for the gate position loop. The cost function is:

$$J_u = \begin{bmatrix} \Phi_{\infty} \end{bmatrix},$$  

(23)

where $\Phi_{\infty}$ is the spectral density of the weighted sum of the tracking error and the control signals, $\phi = P_e F_u$, where $P_e \in R(s)$ and $F_u \in R(s)$ are frequency-dependent dynamic weighting functions, $P_u = P_u / P_d$ and $F_u = F_u / F_d$. The dynamic weighting functions, $P_u, P_d, F_u, F_d \in R[s]$ are the minimum phase and strictly stable polynomials which are chosen to achieve desired specifications and requirements. The optimal controller is calculated by:

$$C_u = (F_u G_u) / \left( P_u H_u \right),$$  

(24)

where $H_u$ and $G_u$ are calculated from a coupled-diophantine equation pair, and $H_u G_u \in R(s)$. The design algorithm [21] and more about the choice of the weighting functions in their parameterised version can be found in [8,21,25,26].
4.2. Outer Multivariable Loop Design

The polynomial multivariable $H_\infty$ design method [5] can be used to take advantage of robustness. The design theory minimizes the following cost function based on the $H_\infty$ norm for the systems with $m$ inputs and $r$ outputs:

$$ J = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \text{trace}(X(s)) ds , $$

(25)

where $X = W_o^*(Q \Phi_o + G \Phi_m + \Phi_u \Phi_o G_o^* + R \Phi_m W_o)$ and $W_o$ is minimum phase stable real rational robustness weighting function $W_o = R_o A_o^\infty \in R^{mm}(s)$. $Q$ is the error weighting function, $Q_e = P_e' P_e$, $Q_i \in R^{rr}(s)$, $R_e$ is the control weighting function, $R_e = F_e' F_e$, $R_e \in R^{mm}(s)$, $G_c$ is the cross-weighting function, $G_c = P_c' F_c$, $G_e \in R^{mr}(s)$, $\Phi_o$ is the error signal spectrum, $\Phi_m$ is the control signal spectrum, $\Phi_m$ and $\Phi_m$ is the spectrum between the error and control signals, and vice versa.

The outer loop is a typical single-input multi-output (SIMO). The full multivariable theory [5] was modified to be used in designs of the SIMO systems [21]. The modified method was applied to excitation control and governor control problems and significant improvements were obtained [5,21,22,25,26]. The real rational dynamic weighting functions $P_c$ and $F_c$ can be parameterized to shape the closed loop response and to achieve the design requirements as:

$$ P_c = P_{cm} P_{cd}^{-1} = \frac{P_{cm} P_{cm1} \ldots P_{cmr}}{P_{cd1} P_{cd2} \ldots P_{cmr}} , $$

(26)

$$ F_c = \frac{F_{cm}}{F_{cd}} = \rho \frac{(s + k_{e1})}{(k_{e2}s + k_{e2})} , $$

(27)

where $P_c \in R^{mr}(s)$, $F_c \in R[s]$, $P_{cm} \in R^{mr}(s)$, $P_{cd} \in R^{rr}(s)$, $P_{cmi} (s) = s + k_{e1}$, $P_{cdi} (s) = k_{e2}s + \epsilon_i$, $P_{cmi} (s), P_{cmi} (s) \in R[s]$, and $k_{e1}$, $k_{e2}$, $k_{e11}$, $k_{e21}$, $k_{e22}$, $\rho$, $\epsilon_i$ are tuning parameters. The available software has been developed to a level where these parameters are the design engineer’s entries to the Matlab M-file to perform the $H_\infty$ design. Then, the real rational optimal controller obtained in Youla form is given by

$$ C_o = \frac{F_{oo} C_{o1} F_{oo} C_{o2} \ldots F_{oo} C_{or} P_{oo} C_{oe}}{P_{oo} C_{o1} P_{oo} C_{o2} \ldots P_{oo} C_{oe}} , $$

(28)

where $C_{oe}$ and $C_{oe}$ are strictly stable polynomials. Some guidelines on how to select these weighting functions were given in reference [5] and extensive information was given in reference [25,26]. The main issue is that the dynamic weighting elements may be chosen to penalise the error and control signal in specific (low and mid-frequency) frequency ranges. In general, integral action, high frequency roll-off, desired gain of controllers and bandwidth are obtained if the dynamic weighting functions, Equation (26) and Equation (27), and tuning parameters are chosen properly.

4.3. Objectives for the Controllers

Every system has its own performance requirements and specifications that are usually given in terms of open- and closed-loop time- and frequency-domain responses. Since the governor system has multiple loops to be designed, each loop needs different measures. For the controllers: (i) high frequency roll-off property is needed in all controllers to attenuate shaft torsional oscillations, measurement noise and some other high frequency modes; low gain in high frequency region is obtained by this property; (ii) the controller, $C_s$, should include integral action for zero steady-state speed error; (iii) the integral action is not needed in the controllers, $C_a$, $C_e$ unless these introduce significant steady-state speed error; (iv) the transient performance could be improved by the controllers, $C_e$, $C_s$, $C_a$; (v) the robust design should be based on the nominal plant parameters, and the stability robustness test should be performed in order to make the system insensitive to the plant parameter changes; and (vi) the time and frequency domain characteristics of the compensated system should be reasonable to ensure the stability of the system; for example, overshoot is very important and not desired for such systems [17,19].

The robust governor design procedure is as follows:
Step 1: Find the range of the plant parameters.

Step 2: Choose the nominal plant parameters and decide the uncertainties.

Step 3: Choose the design theory dynamic weighting functions given in Section 4.1.

Step 4: Use the SISO $H_\infty$ design algorithms given in [21] to calculate the optimal controller, $C_{in}$. Then, check the requirements for the gate position loop to be satisfied. Otherwise go to step 3 and change the tuning parameters until the requirements are achieved. (Note: Consider cascade inner controller design rules to design $C_{in}$. These rules are given in [8,26]).

Step 5: Choose the tuning parameters of the dynamic weighting functions given in Equation (26) and Equation (27).

Step 6: Use the SIMO $H_\infty$ design algorithms given in [21] to calculate the optimal controllers, $aC$, $bC$, and $cC$, simultaneously. Then, check the overall open- and closed-loop system requirements and specifications to be satisfied. Perform the stability robustness test (Equation 12) and the disturbance rejection test (Equation 14) to be satisfied. Otherwise, go to step 5 and change the tuning parameters until the desired robustness and specifications are achieved.

The weighting functions and the calculated controller for the gate position loop is

$$P_{in} = 1, \quad P_{cd} = 0.7s + 10^{-10}, \quad F_{cd}(s) = 1, \quad F_{cd}(s) = 1, \quad C_{in} = \frac{0.39s + 1}{0.7s}.$$  

A first order controller is obtained and integral action is included to eliminate the error at the gate position. The closed-gate position loop occurs at the third–order transfer function and the dominant time constant is 0.15 s which is very smaller than the outer open-loop plant dominant time constant (2 s). The time constants are calculated using the nominal plant parameters. The weighting functions for the outer multivariable-loop design are chosen as:

$$F_c = -\frac{0.6(s + 5)}{1}, \quad P_c = \left[\begin{array}{c} 1 \\ 0.1s + 10^{-10} \\ 1 \end{array}\right].$$

The calculated controllers, $aC$, $bC$, and $cC$, are given in the Appendix. As expected, high frequency roll-off is obtained in all controllers and integral action is included in the controller, $cC$. The gains of the controllers are very small at high frequencies and this improves rejection of the high frequency disturbances and that of the torsional oscillations. The plant is 8th order, the calculated controllers are 7th and 6th orders. The order of a controller is always around the order of the plant in $H_\infty$ the designs, and this is demonstrated in several studies [5,21,25,26]. The classical controllers (PI and PID) are calculated using the modified version of Ziegler-Nichols' Quarter Amplitude method for low overshoots and are given in the Appendix.

5. SIMULATION RESULTS

Time- and frequency-domain responses were obtained in simulations to investigate the effectiveness of the proposed robust design. All these indicated that the robust governor provides the required stability and performance specifications. Frequency-response characteristics allow good insight into the tuning of the control systems when compared to time-domain responses [13]. The results show that gain and phase margins are significantly improved such that 37.9 dB gain margin and 85.3° phase margin was obtained frequency response of the open-loop system as illustrated in Figure 2. It is demonstrated that the phase margin is significantly improved at the critical frequency of inter-area modes between 1.5 and 6 rad/s. On the other hand, 7.7 dB and 9.9 dB gain margins for the PI and PID designs were obtained, respectively. The higher bandwidth was also obtained in the present design, 1.61 rad/s, while it was 0.55 rad/s for the PI and 0.18 rad/s for PID designs. The bandwidths were calculated using the closed-loop system transfer function. The larger bandwidth denoted the larger system operation range. The specifications are presented in Table 1.

Time domain responses were illustrated in Figure 3 for a step set-point speed signal change. The specifications are presented in Table 2. Better results were obtained in $H_\infty$ design such that the overshoot is significantly improved, 0.1% while it was 26.65% for the PI and 34.91% for PID. Hence, classical designs are not acceptable since such large overshoots upset the operation of the turbine.
Figure 2. Frequency response of the compensated open-loop system.

Figure 3. Responses of the system to a step set-point signal change.
5.1 Robustness Test

The maximum and minimum values of the plant parameters were used to obtain the level of the present uncertainty. The robustness test given in Equation (12) was performed and the frequency response was illustrated in Figure 4 such that the maximum value is 0.808 in magnitude, which satisfies the relation given in Equation (12).

Table 1. Frequency Domain Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>PI</th>
<th>PID</th>
<th>$H_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Margin</td>
<td>7.7 dB</td>
<td>9.3 dB</td>
<td>37.9 dB</td>
</tr>
<tr>
<td>Gain cross Frequency</td>
<td>0.22 r/s</td>
<td>0.128 r/s</td>
<td>0.126 r/s</td>
</tr>
<tr>
<td>Phase margin</td>
<td>39°</td>
<td>41°</td>
<td>85.3°</td>
</tr>
<tr>
<td>Phase cross Frequency</td>
<td>0.57 r/s</td>
<td>1.67 r/s</td>
<td>7.07 r/s</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>0.55 r/s</td>
<td>0.18 r/s</td>
<td>1.61 r/s</td>
</tr>
</tbody>
</table>

Table 2. Time Domain Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>PI</th>
<th>PID</th>
<th>$H_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay time</td>
<td>2.55 s.</td>
<td>2.18 s.</td>
<td>2.54 s</td>
</tr>
<tr>
<td>Rise time</td>
<td>3.58 s.</td>
<td>7.2 s.</td>
<td>2.1 s</td>
</tr>
<tr>
<td>Overshoot</td>
<td>26.65%</td>
<td>34.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Settling time (1%)</td>
<td>25 s.</td>
<td>87 s.</td>
<td>6.32 s</td>
</tr>
</tbody>
</table>

Figure 4. Response of the robustness test.
5.2. Load Responses

Load test is an important criterion [7]. A step load change can provide an indication about stability of the system [2]. A step load change of 0.05 in magnitude \( \Delta L(t) = 0.05 \) was applied to the system and the time-domain responses were illustrated in Figure 5 such that the robust design provided good load rejection. The magnitude of 0.02 peak value was obtained in speed variations in robust design, and the transient signal damped around 30 s. Figure 6 shows the gate position deviations for a \( \pm 0.05 \) square wave load disturbance \( \Delta L(t) \) changes. The steady-state value of the gate position was obtained to be 0.5 in magnitude, because the generator damping term used in present design was \( D = 0.5 \). A smooth gate position signal was obtained, which is a good indication for performance. The spikes during the load changes occurred because of the plant non-minimum phase behavior. The transient characteristics of hydro turbines may be also determined by the water flow in the penstocks [9]. The water disturbance, a 0.01 step variation in magnitude from \( d_{i} \) was applied to the system. The time-domain responses were illustrated in Figure 7. An excellent response was obtained in the robust design so that the disturbance rejection measure is \( \| \Delta \omega / d_{i} \| < 0.0012 \) while a value of 0.005 is obtained for the PI design and 0.0056 for PID designs.

![Figure 5. System responses to a 0.05 step output load disturbance.](image)

6. CONCLUSIONS

In the present paper, a new robust governor design using a multivariable-cascade control approach was presented for hydro turbine controls to improve the system performance. It was demonstrated that the proposed method is capable of providing a mechanism to deal with the varying system dynamics.

The non-linear turbine model was obtained that included the effects of water hammer, travelling waves, inelastic water penstocks. The multivariable-cascade control design procedure was explained step by step to achieve a robust design. Polynomial robust SISO and SIMO \( H_{\infty} \) design methods were employed in the design of the robust governor such that the SISO \( H_{\infty} \) design method was used in the design of the inner loop, while the SIMO \( H_{\infty} \) design method was used in the design of the outer multivariable loop. Plant parameter uncertainties and the stochastic nature of the disturbances were discussed in the robust design to be included in the design procedure. Use of the frequency-dependent
dynamic weighting functions in controller designs were described such that these weighting functions were parameterized to introduce some control functions into the controller and the systems, such as integral action, high frequency roll-off, bandwidth and the gains of the robust controllers in a specific frequency region. The tuning parameters were linked to the control functions.

Figure 6. Gate position for ±0.05 square wave load changes.

Figure 7. System responses to a 0.01 step water disturbance.
The features and nature of the solution method was shown in a case study. Data that were obtained in literature for the case study was used to demonstrate the use of proposed robust governor design using the multivariable-cascade control approach. The design was achieved in two steps such that the inner loop was designed first, and then the outer multivariable loop was designed. The robust governor provided a stable closed loop system even in the presence of uncertainties. The simulation results were illustrated in both time and frequency domains. It was demonstrated that performance of the system was significantly improved by the proposed robust design approach.

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REFERENCES

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APPENDIX

\[ P_I = 1.518 \left(1 + \frac{1}{7.479 s} \right), \quad \text{PID} = 0.675 \left(1 + \frac{1}{4.487 s} + 2.99 s \right) \]

\[ C_s(s) = \frac{0.008 s^5 + 0.0031 s^4 + 0.0463 s^3 + 0.135 s^2 + 0.1097 s + 0.0054}{10 s^6 + 11.3 s^4 + 600.1 s^3 + 1570.2 s^2 + 2620.8 s + 2765.1} \]

\[ C_s(s) = \frac{25 s^5 + 228 s^4 + 1945 s^3 + 6791 s^2 + 6246 s + 351}{s(s^2 + 11.9 s^2 + 105.1 s^3 + 610.6 s^3 + 1604 s^2 + 2664.8 s + 2847.8)} \]

\[ C_s(s) = \frac{0.0786 s^5 + 0.313 s^4 + 4.6338 s^3 + 13.498 s^2 + 10.97 s + 0.5446}{10 s^6 + 11.5 s^5 + 102.3 s^4 + 600.6 s^3 + 1566.3 s^2 + 2618.9 s + 2761.4} \]

Perturbed turbine transfer function:

\[ G_t(s) = \frac{-s^3 + 0.997 s^2 - 39.478 s + 9.8413}{0.5 s^3 + 0.997 s^2 + 19.739 s + 9.8837} \]

The nominal parameters used in design:

\[ T_p = 0.02 \ s, \ \ T_x = 0.5 \ s, \ \ k_p = 0.0003042 \ m/(m^3/s)^2, \ \ \Omega_p = 0.00286, \ \ T_o = 0.4227 \ s, \ \ T_w = 1.5843 \ s, \ \ Q_o = 71.43 \ m^3/s, \ \ L = 465 \ m, \ \ a, \ \ D = 0.5, \ \ H = 4. \]

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