ANALYTICAL SOLUTION FOR MUTUAL COUPLING IN MICROSTRIP PATCH ANTENNA ARRAYS

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الخلاصة:

نقدم فى هذا البحث العلاقات الخاصة بشكل الإشعاع الناتج عن وحدة الهوائي المسطح. وقد افترضنا للحصول على تلك العلاقات أن دالة توزيع التيار هى جا(س)، وثابتة على عرض الوحدة. كما اعتبرنا أيضاً أن عرض الوحدة أقل بكثير من طولها. وتتضمن العلاقات التى حصلنا عليها تأثير وحدات الهوائيات المتجاورة على بعضها بعضاً عند تغير المسافة بينها. ويتضمن البحث إضافة إلى ذلك تأثير علاقات وحدات الهوائيات المتجاورة على الشكل الإجمالي للإشعاع لمنظومة الهوائيات، وذلك عندما تكون المسافة بين الوحدات المختلفة صغيرة.

كما تؤكد نتائج هذا البحث أهمية أخذ تأثير الوحدات المتجاورة على بعضها بعضاً في الاعتبار عند تصميم منظومات الهوائيات للحصول على أشكال مطلوبة للإشعاع.

ABSTRACT

The far-field radiation pattern expressions for a microstrip patch element with sinusoidal current distribution along its length (L) and constant current distribution along its width (W) are presented, assuming that the width of the patch is very small compared to its length. Analytical expressions for mutual coupling between the patch elements are derived and effect of the elemental spacing on the mutual coupling between the microstrip elements is presented. The far-field radiation pattern of the microstrip antenna array is studied taking these mutual coupling effects into account. In an array where the individual patch elements lie in close proximity to each other, the mutual coupling between the array elements affects the resultant far-field radiation. This paper stresses the importance of taking the mutual coupling effects into account and the findings show substantial variation in the resultant far-field radiation pattern of the array due to mutual coupling effects.

Key Words: Antenna arrays, Mutual impedance, Microstrip antennas, Mutual coupling.

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1. INTRODUCTION

Deschamps first proposed the concept of microstrip radiators as early as 1953 [1,2]. However, these arrays were not fabricated as better theoretical models and photo-etching techniques for copper or gold-clad dielectric substrates with a wide range of dielectric constants had not been developed. The first practical microstrip antenna was built in 1970 by Howell and Munson [3,4].

Later, microstrip antenna arrays became very popular for their low profile and light weight as well as their flexibility. Daniel *et al.* [5] discussed the flexibility of the printed technology and offered the possibilities of innovative radiating structures well suited to various applications.

Chen [6] and others studied mutual coupling effects in microstrip patch phased linear antenna array; results were presented with and without mutual coupling. They concluded that mutual coupling can also be the cause of blind scan angles, which should always be avoided within the scan range of the phased array. In this paper, we presented mutual coupling effects in the planar microstrip antenna arrays, deriving the expressions analytically.

For a typical microstrip patch element (shown in Figure 1), the following assumptions were made in studying the far-field radiation of the microstrip throughout this paper.

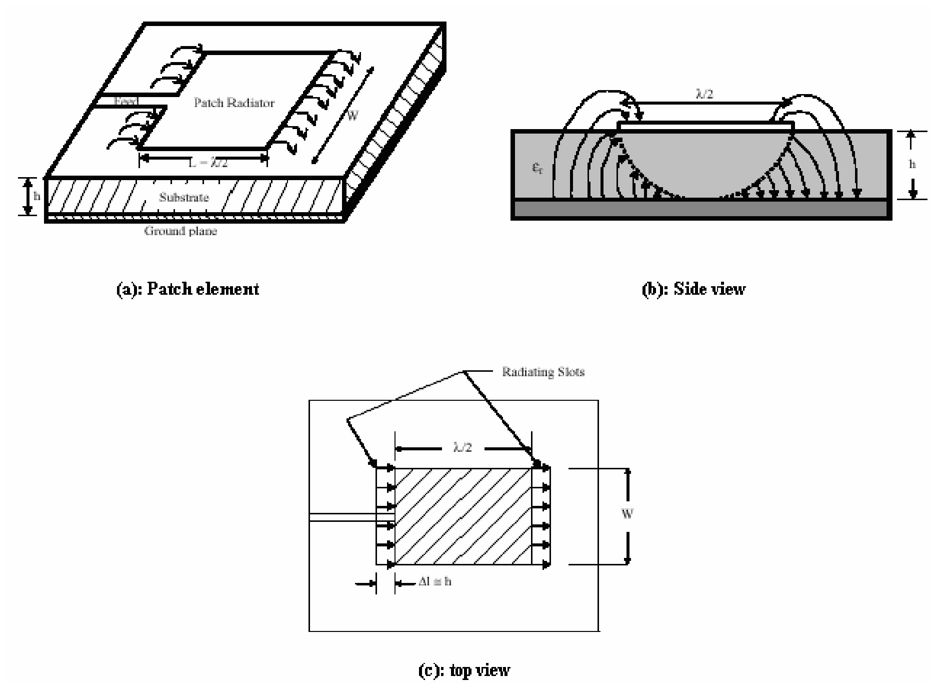

Figure 1. Microstrip patch element.

- The length of the microstrip patch element 'L' is very large compared to its width 'W'.
- The current distribution for the patch element varies along the length of the patch and is constant along its width.
- The variation of the current along the length of the microstrip patch is approximated by a sinusoidal distribution.
- The length of the microstrip patch is $\lambda/2$ where 'λ' is the operating wavelength.
- The microstrip patch element is located in free space.

This paper presents analytical expressions for mutual coupling based on these assumptions and emphasizes the effect of the mutual coupling on the resultant far-field radiation pattern of the microstrip antenna array.

Radiation from a microstrip patch element can be understood by considering a simple case where a rectangular patch is spaced a small fraction of a wavelength above a ground plane. Assuming that field varies only along the patch length, which is half a wavelength ($\lambda/2$), radiation may be ascribed mostly to the fringing fields at the open-circuited edges of the patch. The fields at the end can be resolved into normal and tangential components with respect to the ground plane. The normal components are 180° out of phase because the patch line is $\lambda/2$ long; therefore the far fields produced by them cancel in the broadside direction. The tangential components (those parallel to the ground plane) are in phase, and the resulting fields combine to give maximum radiated field normal to the surface of the structure; *i.e.*, the broadside direction [7,8].

Far ***E*** and ***H*** fields can be calculated from the vector potential ***A***, which is a function of the current distribution of the microstrip patch. However, in an array where the elements lie close to each other, the coupling effects change the effective current distributions on the individual elements and must be taken into consideration while calculating the far-field electromagnetic radiation of the microstrip patch element.

In this study we present the far-field radiation assuming the current variation is only along the length of the patch [1]. Mutual coupling effects were taken into consideration by applying the analytical expressions for the near fields and mutual impedances to this microstrip array case. The results are presented for an 8×8 element patch antenna array where the patterns are compared in case of mutual coupling to the case where mutual coupling is ignored.

2. FAR FIELD EXPRESSIONS OF THE MICROSTRIP PATCH ANTENNA

For the microstrip patch element shown in Figure 2 with sinusoidal current variation along the length of the patch and no variation along the width, the expressions for the current distribution and the far-field expression are given by Equations (1–7) [1].

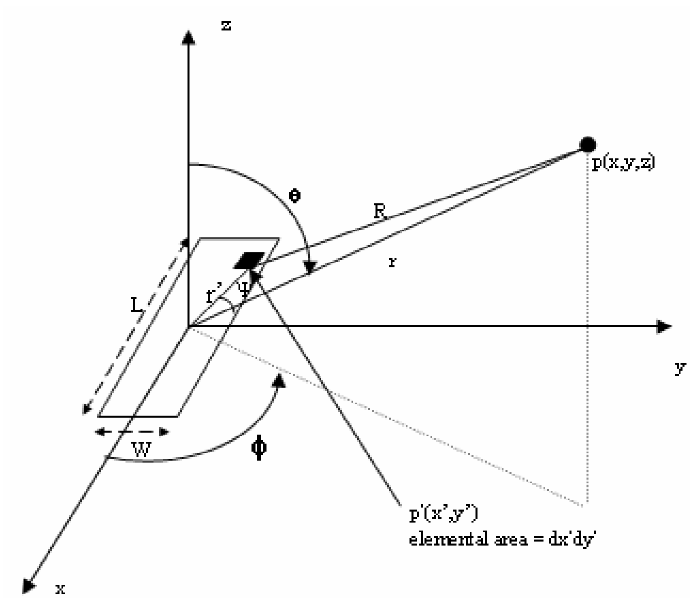

Figure 2. Microstrip patch located in xy-plane with origin at the center. p(x,y,z) is an arbitrary point in space and p′(x′,y′) is an arbitrary point on the microstrip patch.

The microstrip patch is in the *XY*-plane and the *Z*-axis is normal to the patch. The center of the microstrip is located at the origin. The spherical coordinate system is shown in Figure 2 where ϕ is the azimuth angle and θ is the elevation angle. Point $p'(x',y')$ represents any point on the patch. The surface current density at that point is given by:

$$J(x',y') = J(x') = \begin{cases} \bar{a}_x \dfrac{I_0}{W} \sin\left(k\left(\dfrac{L}{2} - x'\right)\right) & 0 \le x' \le L/2 \\ \bar{a}_x \dfrac{I_0}{W} \sin\left(k\left(\dfrac{L}{2} + x'\right)\right) & -L/2 \le x' \le 0 \end{cases} \tag{1}$$

$$E_\theta = -j\eta I_0 \frac{\exp(-jkr)}{\pi r} \frac{\sin\left(\frac{kW}{2}\sin\theta\sin\phi\right)}{kW\tan\theta\tan\phi} \frac{\left[\cos\left(\frac{kL}{2}\sin\theta\cos\phi\right)-\cos\left(\frac{kL}{2}\right)\right]}{\left(1-\sin^2\theta\cos^2\phi\right)} \tag{2}$$

$$E_\phi = j\eta I_0 \frac{\exp(-jkr)}{\pi r} \frac{\sin\left(\frac{kW}{2}\sin\theta\sin\phi\right)}{kW\sin\theta} \frac{\left[\cos\left(\frac{kL}{2}\sin\theta\cos\phi\right)-\cos\left(\frac{kL}{2}\right)\right]}{\left(1-\sin^2\theta\cos^2\phi\right)} \tag{3}$$

$$H_\theta = -jI_0 \frac{\exp(-jkr)}{\pi r} \frac{\sin\left(\frac{kW}{2}\sin\theta\sin\phi\right)}{kW\sin\theta} \frac{\left[\cos\left(\frac{kL}{2}\sin\theta\cos\phi\right)-\cos\left(\frac{kL}{2}\right)\right]}{\left(1-\sin^2\theta\cos^2\phi\right)} \tag{4}$$

$$H_\phi = -jI_0 \frac{\exp(-jkr)}{\pi r} \frac{\sin\left(\frac{kW}{2}\sin\theta\sin\phi\right)}{kW\tan\theta\tan\phi} \frac{\left[\cos\left(\frac{kL}{2}\sin\theta\cos\phi\right)-\cos\left(\frac{kL}{2}\right)\right]}{\left(1-\sin^2\theta\cos^2\phi\right)} \tag{5}$$

where

$$\eta = \frac{\omega\mu}{k} \quad (\textit{for free space})$$

therefore, the total Electric and Magnetic fields are given by

$$E = \sqrt{(E_\theta)^2+(E_\phi)^2} \tag{6}$$

$$H = \sqrt{(H_\theta)^2+(H_\phi)^2} \tag{7}$$

Equations (2–7) are used to find the far-field radiation pattern of the patch element. This element pattern is then multiplied by an 8×8 element array factor to find the far-field radiation pattern of the microstrip array. The resultant expression is the far-field radiation pattern of the microstrip antenna array without coupling. The radiation pattern without mutual coupling is then compared with our results obtained after taking mutual coupling into account as shown in later sections.

3. MUTUAL COUPLING BETWEEN TWO MICROSTRIP PATCH ANTENNAS

When two patch elements are brought in the vicinity of each other, the current in each element changes in both amplitude and phase. The amount of change depends on the mutual coupling between the elements. Since the currents are changed, the far-fields due to these elements change. To find the resultant far-field radiation pattern, the variation in the phase and amplitude of the currents in these two individual patch elements are calculated first and then the resultant far-field pattern can be found. Self and mutual impedances can be derived from the near field calculations. These impedance expressions can be used to find the effective current distributions on the individual patch elements. The current distributions are then used for calculating the far-field which is the required field in the presence of mutual coupling. To find both self and mutual impedance, consider the arrangement of the two patches shown in Figure 3.

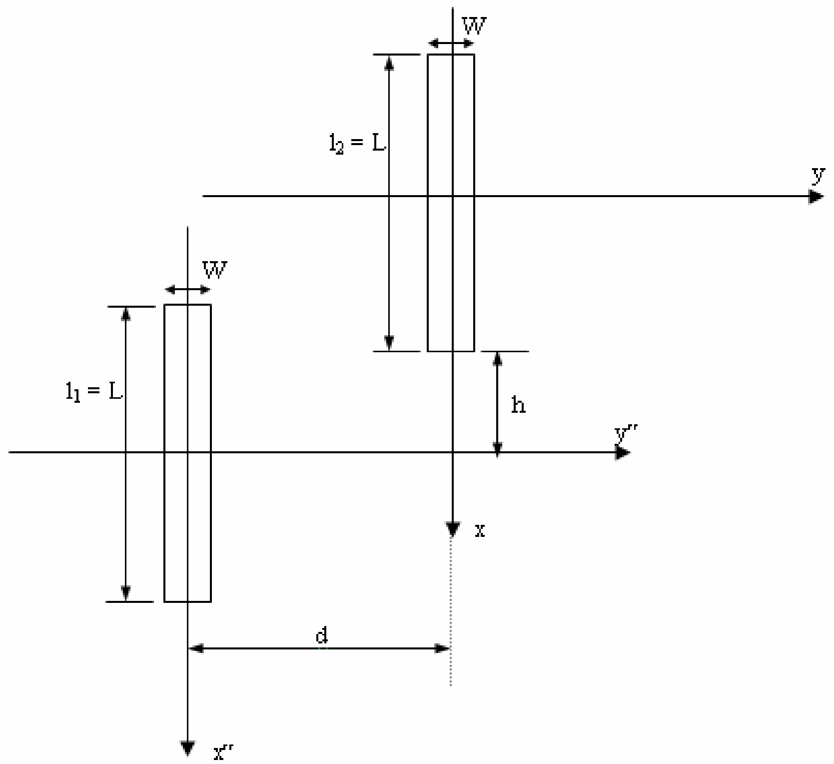

Figure 3. Microstrip patch elements positioning for mutual coupling calculations.

The near fields can be calculated using the vector potential equation (8) [1].

$$A = \frac{\mu}{4\pi}\iint_s J(x', y')\frac{\exp(-jkR)}{R}ds' \qquad (8)$$

where

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \qquad (9)$$

The complexity lies in evaluating this integral, for which only the plane of the microstrip element is considered, as

$$H_\theta = \frac{1}{4\pi}\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}\frac{\partial}{\partial\rho}\left(J(x', y')\frac{e^{-jkR}}{R}\right)dx'dy' \qquad (10)$$

we require only the fields in the plane of the microstrip patch for mutual coupling calculations [1]. Only the A_x component exists as the current variation is only along the *x*-direction. Using Maxwell's equation, we can find the *E* and *H* fields from the vector potential ***A***. The near fields are calculated in cylindrical coordinates. It can be deduced that only the H_θ component exists and is given by Equation (10).

Equation 10 can be transformed to the following equation.

$$H_\theta = \frac{I_0}{4\pi jW}\int_{-W/2}^{W/2}\frac{1}{(y-y')}\left[e^{-jkR_1} + e^{-jkR_3} - 2\cos\left(\frac{kL}{2}\right)e^{-jkR_2}\right]dy' \qquad (11)$$

To evaluate Equation (11), we make use of the formulae given in tables book [10] therefore consider the following equation.

$$\int_{U_1}^{L_1} \frac{R_1 e^{-jkR_1}}{R_1^{\,2} - (x+L/2)^2} dR_1 = \frac{1}{2}\int_{U_1}^{L_1} \frac{e^{-jkR_1} dR_1}{R_1 + (x+L/2)} + \frac{1}{2}\int_{U_1}^{L_1} \frac{e^{-jkR_1} dR_1}{R_1 - (x+L/2)}$$

$$= \frac{1}{2} e^{jk(x+L/2)} \left[Ei(-jk(L_1 + x + L/2) - Ei(-jk(U_1 + x + L/2) \right] +$$

$$+ \frac{1}{2} e^{-jk(x+L/2)} \left[Ei(-jk(L_1 - x - L/2) - Ei(-jk(U_1 - x - L/2) \right]$$

therefore Equation (11) reduces to the following equation (12).

$$H_\theta = \frac{I_0}{8\pi jW} \left\{ \begin{array}{l} e^{jk(x+L/2)} \left[Ei[-jk(L_1 + x + L/2)] - Ei[-jk(U_1 + x + L/2)] \right] \\ + e^{-jk(x+L/2)} \left[Ei[-jk(L_1 - x - L/2)] - Ei[-jk(U_1 - x - L/2)] \right] \\ + e^{jk(x-L/2)} \left[Ei[-jk(L_3 + x - L/2)] - Ei[-jk(U_3 + x - L/2)] \right] \\ + e^{-jk(x-L/2)} \left[Ei[-jk(L_3 - x + L/2)] - Ei[-jk(U_3 - x + L/2)] \right] \\ - 2\cos\left(\frac{kL}{2}\right) e^{jkx} \left[Ei[-jk(L_2 + x)] - Ei[-jk(U_2 + x)] \right] \\ - 2\cos\left(\frac{kL}{2}\right) e^{-jkx} \left[Ei[-jk(L_2 - x)] - Ei[-jk(U_2 - x)] \right] \end{array} \right\} \quad (12)$$

$$E = \frac{1}{j\omega\varepsilon} (\nabla \times H)$$

$$E = E_x = \frac{1}{j\omega\varepsilon} \left(-\frac{1}{\rho} \frac{\partial(\rho H_\theta)}{\partial \rho} \right)$$

$$E_x = -\frac{1}{j\omega\varepsilon y} \frac{\partial(y H_\theta)}{\partial y} \qquad (as\ \rho = y)$$

$$E_x = \left\{ \begin{array}{l} \dfrac{\eta}{4\pi}\dfrac{I_0}{W}\dfrac{1}{k}\left[\begin{array}{l} \dfrac{(y+W/2)}{L_1{}^2-(x+L/2)^2}e^{-jkL_1} - \dfrac{(y-W/2)}{U_1{}^2-(x+L/2)^2}e^{-jkU_1} \\ + \dfrac{(y+W/2)}{L_3{}^2-(x-L/2)^2}e^{-jkL_3} - \dfrac{(y-W/2)}{U_3{}^2-(x-L/2)^2}e^{-jkU_3} \\ -2\cos\left(\dfrac{kL}{2}\right)\left(\dfrac{(y+W/2)}{L_2{}^2-x^2}e^{-jkL_2} - \dfrac{(y-W/2)}{U_2{}^2-x^2}e^{-jkU_2}\right) \end{array} \right] \\ + \dfrac{j\eta}{ky}H_\theta \end{array} \right\} \quad (13)$$

where

$$U_1 = \sqrt{(x+L/2)^2+(y-W/2)^2} \quad ; \quad L_1 = \sqrt{(x+L/2)^2+(y+W/2)^2}$$
$$U_2 = \sqrt{x^2+(y-W/2)^2} \quad ; \quad L_2 = \sqrt{x^2+(y+W/2)^2}$$
$$U_3 = \sqrt{(x-L/2)^2+(y-W/2)^2} \quad ; \quad L_3 = \sqrt{(x-L/2)^2+(y+W/2)^2}$$

$$Z_{21i} = \frac{V_{21i}}{I_{1i}} = -\frac{1}{I_{1i}I_{2i}}\int_{-l_2/2}^{l_2/2} E_{x21}(x)I_2(x)dx \quad (14)$$

$$Z_{21m} = Z_{21i} \times \frac{I_{1i}}{I_{1m}}\frac{I_{2i}}{I_{2m}}$$ (expression for mutual impedence)

For identical patches, the above Equation (15) can be written as

$$Z_{21m} = -\frac{\eta}{4\pi}\frac{1}{W}\frac{1}{k}\int_{-L/2}^{L/2}\left\{\sin\left(k\left(\frac{L}{2}-|x|\right)\right)p(x)\right\}dx - \frac{j\eta}{Wky''}\int_{-L/2}^{L/2}\left\{\sin\left(k\left(\frac{L}{2}-|x|\right)\right)q(x)\right\}dx$$

where

$$p(x) = \frac{(y''+W/2)}{L_1{}^2-(x''+L/2)^2}e^{-jkL_1} - \frac{(y''-W/2)}{U_1{}^2-(x''+L/2)^2}e^{-jkU_1} + \frac{(y''+W/2)}{L_3{}^2-(x''-L/2)^2}e^{-jkL_3} - \frac{(y''-W/2)}{U_3{}^2-(x''-L/2)^2}e^{-jkU_3} - 2\cos\left(\frac{kl}{2}\right)\left[\frac{(y''+W/2)}{L_2{}^2-x''^2}e^{-jkL_2} - \frac{(y''-W/2)}{U_2{}^2-x''^2}e^{-jkU_2}\right]$$

$$q(x) = \frac{1}{8\pi j}\left\{ \begin{array}{l} e^{jk(x-h)}\left[Ei[-jk(L_1+x-h)] - Ei[-jk(U_1+x-h)]\right] \\ + e^{-jk(x-h)}\left[Ei[-jk(L_1-x+h)] - Ei[-jk(U_1-x+h)]\right] \\ + e^{jk(x-h-L)}\left[Ei[-jk(L_3+x-h-L)] - Ei[-jk(U_3+x-h-L)]\right] \\ + e^{-jk(x-h-L)}\left[Ei[-jk(L_3-x+h+L)] - Ei[-jk(U_3-x+h+L)]\right] \\ -2\cos\left(\dfrac{kL}{2}\right)e^{jk(x-h-L/2)}\left[\begin{array}{l} Ei[-jk(L_2+x-h-L/2)] \\ -Ei[-jk(U_2+x-h-L/2)] \end{array}\right] \\ -2\cos\left(\dfrac{kL}{2}\right)e^{-jk(x-h-L/2)}\left[\begin{array}{l} Ei[-jk(L_2-x+h+L/2)] \\ -Ei[-jk(U_2-x+h+L/2)] \end{array}\right] \end{array} \right\}$$

$$U_1 = \sqrt{(x''+L/2)^2 + (y''-W/2)^2} \quad ; \quad L_1 = \sqrt{(x''+L/2)^2 + (y''+W/2)^2}$$
$$U_2 = \sqrt{x''^2 + (y''-W/2)^2} \quad ; \quad L_2 = \sqrt{x''^2 + (y''+W/2)^2}$$
$$U_3 = \sqrt{(x''-L/2)^2 + (y''-W/2)^2} \quad ; \quad L_3 = \sqrt{(x''-L/2)^2 + (y''+W/2)^2}$$
$$x'' = x - h - L/2 \quad ; \quad y'' = y + d$$

Substituting $y = 0$ and $\eta = 120\,\pi$

$$Z_{21m} = -\frac{30}{Wk(d+W/2)} I_1 + \frac{30}{Wk(d-W/2)} I_2 - \frac{15}{Wkd} I_3 \qquad (15)$$

where

$$I_1 = \int_{-L/2}^{L/2} \sin\left[k\left(\frac{L}{2} - |x|\right)\right] \times \left[e^{-jkL_1} + e^{-jkL_3} - 2\cos\left(\frac{kL}{2}\right) e^{-jkL_2}\right] dx$$
$$I_2 = \int_{-L/2}^{L/2} \sin\left[k\left(\frac{L}{2} - |x|\right)\right] \times \left[e^{-jkU_1} + e^{-jkU_3} - 2\cos\left(\frac{kL}{2}\right) e^{-jkU_2}\right] dx$$
$$I_3 = \int_{-L/2}^{L/2} \sin\left[k\left(\frac{L}{2} - |x|\right)\right] \times S(x)\, dx$$

$$S(x) = \left\{ \begin{array}{l} e^{jk(x-h)}\left[Ei[-jk(L_1+x-h)] - Ei[-jk(U_1+x-h)]\right] \\ + \; e^{-jk(x-h)}\left[Ei[-jk(L_1-x+h)] - Ei[-jk(U_1-x+h)]\right] \\ + \; e^{jk(x-h-L)}\left[Ei[-jk(L_3+x-h-L)] - Ei[-jk(U_3+x-h-L)]\right] \\ + \; e^{-jk(x-h-L)}\left[Ei[-jk(L_3-x+h+L)] - Ei[-jk(U_3-x+h+L)]\right] \\ - \; 2\cos\left(\frac{kL}{2}\right) e^{jk(x-h-L/2)} \left[\begin{array}{l} Ei[-jk(L_2+x-h-L/2)] \\ -Ei[-jk(U_2+x-h-L/2)] \end{array}\right] \\ - \; 2\cos\left(\frac{kL}{2}\right) e^{-jk(x-h-L/2)} \left[\begin{array}{l} Ei[-jk(L_2-x+h+L/2)] \\ -Ei[-jk(U_2-x+h+L/2)] \end{array}\right] \end{array} \right\}$$

The mutual impedance is given by Equation (15). Similarly, the self impedance expression can be derived using the formulas from [1]. The final expression for the self impedance is given in Equation (16).

$$Z_{11m} = -\frac{1}{I_m^{\,2}} \int_{-L/2}^{L/2} E_{x11} I_1(x) dx \Bigg|_{y \to W/2} \qquad (16)$$

$$E_{x11}(x) = 30 \frac{I_0}{W} \frac{1}{k(y+W/2)} \left\{ e^{-jkL_1} + e^{-jkL_3} - 2\cos\left(\frac{kL}{2}\right) e^{-jkL_2} \right\}$$
$$- 30 \frac{I_0}{W} \frac{1}{k(y-W/2)} \left\{ e^{-jkU_1} + e^{-jkU_3} - 2\cos\left(\frac{kL}{2}\right) e^{-jkU_2} \right\}$$
$$+ 15 \frac{I_0}{W} \frac{1}{ky} \left\{ \begin{array}{l} e^{jk(x+L/2)}\left[Ei[-jk(L_1+x+L/2)] - Ei[-jk(U_1+x+L/2)]\right] \\ + \; e^{-jk(x+L/2)}\left[Ei[-jk(L_1-x-L/2)] - Ei[-jk(U_1-x-L/2)]\right] \\ + \; e^{jk(x-L/2)}\left[Ei[-jk(L_3+x-L/2)] - Ei[-jk(U_3+x-L/2)]\right] \\ + \; e^{-jk(x-L/2)}\left[Ei[-jk(L_3-x+L/2)] - Ei[-jk(U_3-x+L/2)]\right] \\ - \; 2\cos\left(\frac{kL}{2}\right) e^{jkx}\left[Ei[-jk(L_2+x)] - Ei[-jk(U_2+x)]\right] \\ - \; 2\cos\left(\frac{kL}{2}\right) e^{-jkx}\left[Ei[-jk(L_2-x)] - Ei[-jk(U_2-x)]\right] \end{array} \right\}$$

$$U_1 = \sqrt{(x+L/2)^2 + (y-W/2)^2} \quad ; \quad L_1 = \sqrt{(x+L/2)^2 + (y+W/2)^2}$$
$$U_1 = \sqrt{x^2 + (y-W/2)^2} \quad ; \quad L_1 = \sqrt{x^2 + (y+W/2)^2}$$
$$U_1 = \sqrt{(x-L/2)^2 + (y-W/2)^2} \quad ; \quad L_1 = \sqrt{(x-L/2)^2 + (y+W/2)^2}$$

From Figure 3, consider the patch elements are placed side-by-side. Then the value of h, a variable indicated in the figure, will be equal to $-L/2$. For the collinear arrangement, the variable s is equal to $(h-L/2)$. The mutual impedance plot obtained from Equation (15) is shown in results section (Figures (4) and (5) respectively). The plot for the self impedance is obtained from Equation 16 and is shown in Figure 6.

4. FAR FIELD ARRAY PATTERN OF MICROSTRIP PATCH ANTENNA ARRAY

The array pattern of a microstrip patch array can be obtained by multiplying the array factor of the array and the element pattern of the microstrip patch. When there is mutual coupling between the closely placed microstrip patch elements, the array factor is calculated taking the mutual coupling into account. Though the expressions are derived for a general $m\times n$ element array, this paper focuses on the results of an 8×8 element array and its far-field radiation patten. Finding the array pattern of the microstrip array is straightforward, using Equation (18), once the resultant current distribution in the microstrip array elements is known after the mutual coupling effect is taken into consideration. For this, we have to first obtain the Z matrix which has mutual and self impedance entries for each array element. Each element of the Z-matrix can be calculated using equations (15) and (16). For a unit input voltage for every microstrip patch in 8×8 element array, the currents in the elements can be found using Equation (17) once the Z-matrix is known. Therefore the far-field radiation pattern for a planar 8×8 element microstrip patch array with mutual coupling can be obtained using Equation (18).

$$V_{mn} = \sum_u \sum_v Z_{mn,uv} I_{uv}$$

where

$Z_{mn,mn}$ are self impedances and

$Z_{mn,uv}$ for $mn \neq uv$ are mutual impedances

$$[I] = [Z]^{-1}[V] \tag{17}$$

$$\begin{aligned} AP_{8\times 8} = \sum_{n=-1}^{-4}\Bigg[&\sum_{m=-1}^{-4} EP\times I_{m,n} \exp\left(-j\left(\left(\frac{2m+1}{2}\right)\Psi_x - ph_{mn}\right)\right) + \\ &\sum_{m=1}^{4} EP\times I_{m,n} \exp\left(-j\left(\left(\frac{2m-1}{2}\right)\Psi_x - ph_{mn}\right)\right)\Bigg] \times \exp\left(-j\left(\frac{2n+1}{2}\right)\Psi_y\right) \\ + \sum_{n=1}^{4}\Bigg[&\sum_{m=-1}^{-4} EP\times I_{m,n} \exp\left(-j\left(\left(\frac{2m+1}{2}\right)\Psi_x - ph_{mn}\right)\right) + \\ &\sum_{m=1}^{4} EP\times I_{m,n} \exp\left(-j\left(\left(\frac{2m-1}{2}\right)\Psi_x - ph_{mn}\right)\right)\Bigg] \times \exp\left(-j\left(\frac{2n+1}{2}\right)\Psi_y\right) \end{aligned} \tag{18}$$

$EP = \sqrt{(E_\theta)^2 + (E_\phi)^2}$ (EP-for field radiation pattern of Microstrip patch element)

The array pattern obtained using Equation (18) is compared with the array pattern obtained without taking mutual coupling into account. The results are shown in the next section.

5. RESULTS

The expressions for the mutual and self impedances are given in Equations (15) and (16) respectively and are used to calculate the Z-matrix of microstrip planar antenna array. After obtaining the Z-matrix, for a given input voltage excitation to the array elements, the resultant current distribution on the elements can be obtained which are then used to find the array pattern. The variation of mutual impedance between two elements placed close to each other is investigated first. This is given for two patch elements placed side-by-side as a function of separation between the centers of the patch elements (d/λ). Similarly the variation of mutual impedance as a function of distance (s/λ) for collinearly placed patches are presented (where $s = h-L/2$). The results are shown in Figures 4 and 5 respectively. The variation of self impedance as a function of its length (L/λ) of the patch is presented in Figure 6.

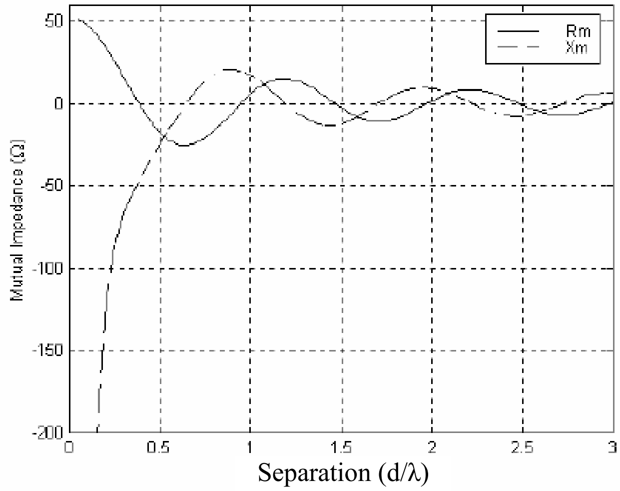

Figure 4. Variation of mutual impedance between two patch elements placed side-by-side as a function of separation between the two elements. From figure 3, h = –L/2 (as both elements lie on the same y-axis).

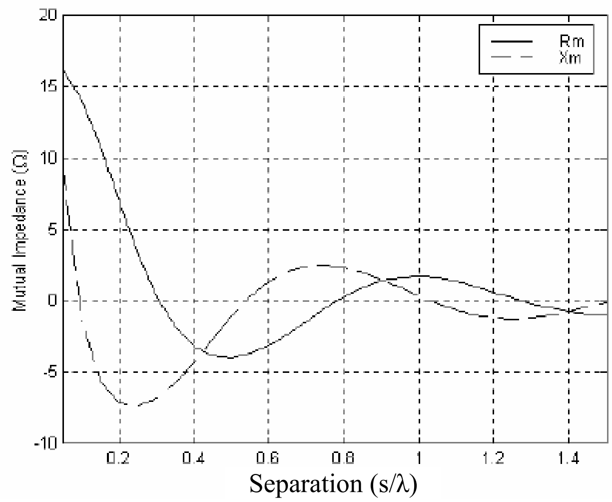

Figure 5. Variation of mutual impedance between two patch elements arranged collinearly as a function of separation between them. From figure 3, s = h –L/2 and both elements lie on the same x-axis. The value of 's' is equal to '0' when 'h' is equal to 'L/2', thereby patch elements touching each other along x-axis.

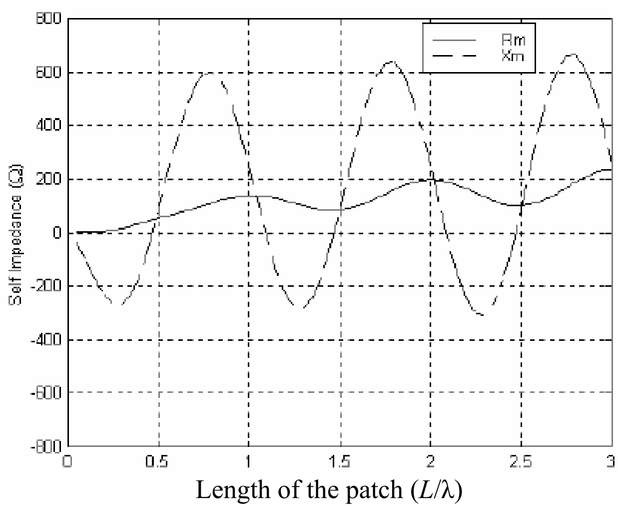

Figure 6. Variation of self impedance (or input impedance) as a function of microstrip patch length.

In this paper, an 8-8 element microstrip array was taken with the length of each patch element as 0.5λ and width 0.01λ. The elements are arranged in *XY*-plane in such a way that their lengths are along the *X*-axis and the separation between the centers of array elements is 0.5λ. Figures 4–6 show that the current distribution is not just due to the input impedance (self-impedance) but also due to the mutual impedance due to the elements arranged in the array. In reference [1], these mutual impedance values were not taken into consideration in the analytical expressions that give rise to the far field radiation. In this paper, we have incorporated these mutual coupling values to find the radiation pattern of the microstrip array analytically.

The far-field radiation pattern of the antenna array in space is displayed in spherical coordinates. The radiation intensity at any point in space can be expressed in terms of (r,θ,ϕ). Therefore the far-field radiation intensity of the microstrip array can be expressed in terms of (θ,ϕ) coordinates (for a constant distance r from the center of the antenna array). The 3D plot showing the radiation intensity can be understood from the schema shown in Figure 7.

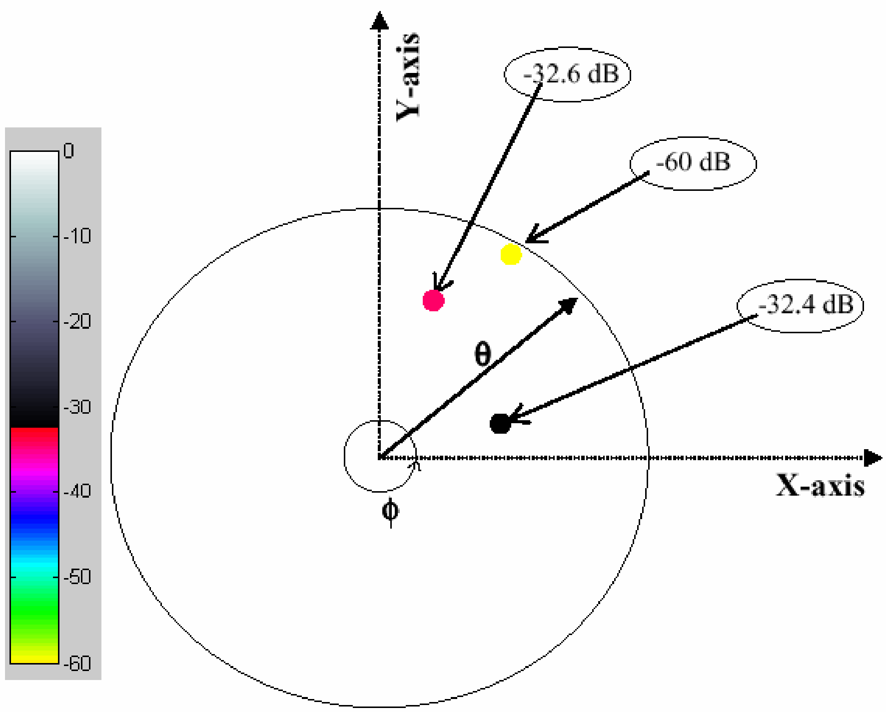

Figure 7. Description of the three dimensional plot that is used for displaying the 3D radiation pattern results.

In Figure 7, any point in *XY*-plane can be represented with (θ,ϕ) spherical coordinates. For any point on *X*-axis, ϕ is equal to 0 or 360 degrees while θ varies from 0 degrees when the point is on the origin to a maximum of 90 degrees when the point is on the circumference of the circle shown in the diagram. The azimuth angle ϕ varies from 0 degrees to a maximum of 360 degrees as indicated in the diagram. The colored spots at any point represent the radiation intensity at those particular points and can be known from the color bar given in the diagram. Therefore the diagram shows the radiation above the *XY*-plane. This schema in figure 7 is used to present the three dimensional radiation results for the 8×8 element microstrip array under discussion where the microstrip array is placed in the *XY*-plane with the patch elements lying along the *X*-axis.

The radiation pattern of the 8×8 element microstrip array is shown in Figure 8 where the mutual coupling effects are neglected. This result is compared with Figure 9, which displays the radiation pattern of the same array taking mutual coupling effects into account.

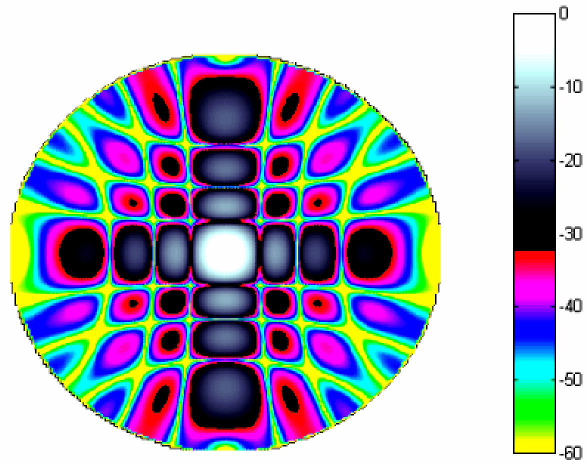

Figure 8. 3D array pattern of the 8×8 element microstrip antenna array after taking mutual coupling effects into account. The microstrip patch element dimensions (L=0.5λ, W=0.01λ), inter-element spacing in both X- and Y- directions is 0.5λ .

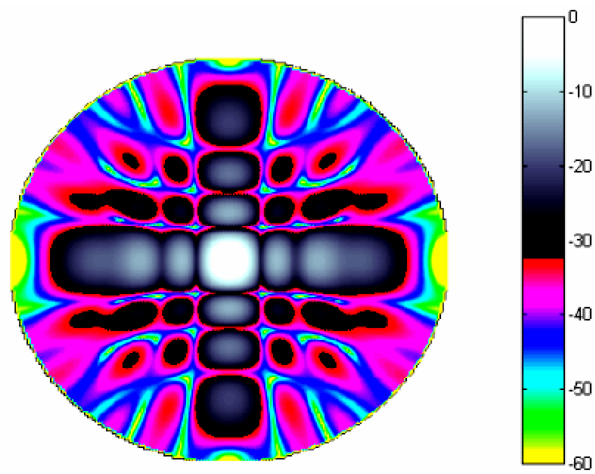

Figure 9. 3D array pattern of the 8×8 element microstrip antenna array without taking mutual coupling effects into account. The microstrip patch element dimensions (L=0.5λ, W=0.01λ), inter-element spacing in both X- and Y- directions is 0.5λ .

From Figures 8 and 9, it can be clearly seen that the radiation pattern has changed considerably due to mutual coupling between the array elements. If we observe the patterns in both Figures 8 and 9, there is more variation along the *X*-axis than along the *Y*-axis. This variation can be further understood from the two dimensional plots taken along the *X*-axis (keeping $\phi = 0^\circ$) and comparing them to see the variation in the radiation pattern due to mutual coupling. This comparison is shown in Figure 10. A similar comparison of the plots is made in Figure 11, taking radiation results along *Y*-axis (keeping $\phi = 90^\circ$). Comparisons (of Figures 10 and 11) show the variation in the pattern due to coupling along the *X*-axis compared to along the *Y*-axis, respectively.

The expressions derived can be used for any number of elements in the planar array and any patch length as long as the width of the patch is small compared to the length and with negligible thickness.

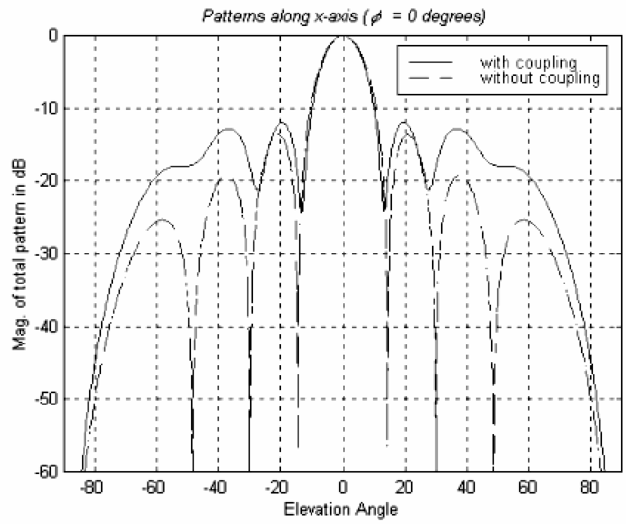

Figure 10. Comparison of the patterns of the 8×8 element microstrip patch antenna array along the X-axis. The figure shows the variation in the radiation pattern due to mutual coupling effects in the antenna array. This is a two dimensional comparision of Figures 8 and 9 along the X-axis.

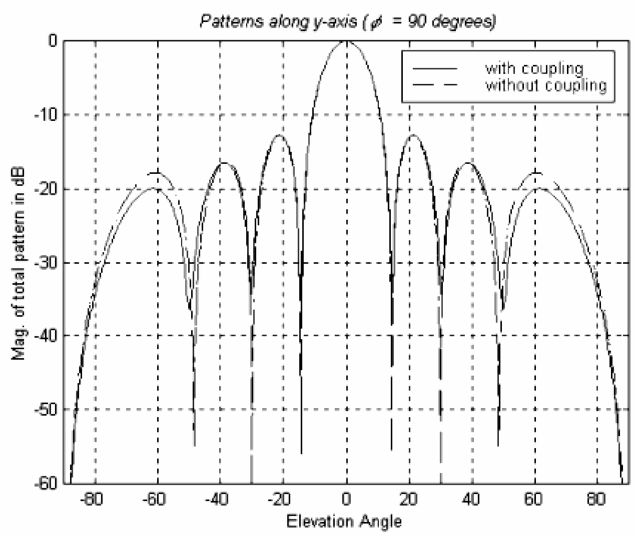

Figure 11. Comparison of the patterns of the 8×8 element microstrip patch antenna array along the Y-axis. The figure shows the variation in the radiation pattern due to mutual coupling effects in the antenna array. This is a two dimensional comparision of Figures 8 and 9 along the Y-axis.

Figures 10 and 11 show that there is greater variation in the pattern along the *X*-axis than along *Y*-axis.

6. CONCLUSION

Mutual impedances between microstrip patch antenna elements in rectangular planar arrays have been investigated. Sinusoidal current distribution along the length of the patch radiator is assumed. Array patterns for an 8×8 planar array of microstrip elements were obtained when the mutual coupling between the array elements was considered. The results show that the mutual coupling between the elements of the microstrip patch array changes the far field pattern considerably and therefore cannot be neglected. The variation in the pattern is more pronounced along the *X*-axis than along the *Y*-axis. These results are important in the design of phased arrays with prescribed main beam and null locations.

7. ACKNOWLEDGMENT

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